

MODELING, IDENTIFICATION, AND CONTROL OF HYSTERETIC SYSTEMS WITH
APPLICATION TO VANADIUM DIOXIDE MICROACTUATORS

By

Jun Zhang

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Electrical Engineering - Doctor of Philosophy

2015

ABSTRACT

MODELING, IDENTIFICATION, AND CONTROL OF HYSTERETIC SYSTEMS WITH APPLICATION TO VANADIUM DIOXIDE MICROACTUATORS

By

Jun Zhang

Hysteresis nonlinearity in magnetic and smart material systems hinders the realization of their potential in sensors and actuators. The goal of this dissertation is to advance methods for modeling, identification, and control of hysteretic systems. These methods are applied to the inverse compensation, self-sensing feedback control, and robust control of vanadium dioxide (VO_2) microactuators. As a novel smart material, VO_2 undergoes a thermally induced insulator-to-metal transition and a structural phase transition, exhibiting pronounced hysteresis in electrical and mechanical domains.

With the goal of obtaining accurate hysteresis models while maintaining a low model complexity, optimal compressions for two popular hysteresis models, namely the Preisach operator and the generalized Prandtl-Ishlinskii (GPI) model, are studied, where the Kullback-Leibler divergence and entropy, respectively, are adopted to quantify the information loss in model compression. While the optimal compression of the Preisach operator is realized using exhaustive search, dynamic programming is employed to optimally compress the GPI model efficiently. Both simulation and experimental results demonstrate that the proposed algorithms yield superior performances than typically adopted schemes.

In order to identify the Preisach operator, existing work involves applying a complicated input sequence and measuring a large set of output data. We propose an efficient approach to identify the Preisach operator that requires fewer measurements. The output of the Preisach operator is transformed into the frequency domain, generating a sparse vector of discrete cosine transform

(DCT) coefficients. The model parameters are reconstructed using a compressive sensing-based algorithm. The effectiveness of the proposed scheme is illustrated through simulation and experiments.

A few new contributions have been made to the modeling and control of VO₂ microactuators. In order to capture the non-monotonic curvature-temperature hysteresis of VO₂ microactuators, physics-motivated models that combine a monotonic hysteresis operator for phase transition induced curvature and a memoryless operator for differential thermal expansion induced curvature are proposed. Effective inverse compensation schemes for the proposed non-monotonic hysteresis models are presented. The modeling and inverse compensation schemes are validated experimentally.

Since external sensing systems are not desirable with micro devices, a self-sensing model is developed for VO₂ microactuators to estimate the deflection from the resistance measurement. We exploit the physical understanding that each of the resistance and the deflection is determined by a hysteretic relationship with the temperature, which is modeled with a GPI model and an extended GPI model, respectively. The self-sensing model is obtained by cascading the extended GPI model with the inverse of the GPI model. The performance of the self-sensing scheme is experimentally evaluated with proportional-integral control. Finally, an H_∞ robust controller is further developed, where a simple polynomial-based self-sensing scheme is adopted, as the emphasis is on accommodating the uncertainties produced by the hysteresis nonlinearity and the self-sensing error. The effectiveness of the proposed approach is demonstrated through experiments.

To Mom, Dad, and my wife Ran

ACKNOWLEDGMENTS

I would like to express my warmest gratitude to my advisor, Prof. Xiaobo Tan, for providing me with the opportunity in Smart Microsystems Lab at Michigan State University (MSU) to pursue a Ph.D. degree. During my doctoral study, he has profoundly inspired and influenced me in many ways with his expertise, enthusiasm, and vision. I am forever grateful for his guidance with encouragement and patience in the fascinating field of smart materials. He has also offered me with invaluable advices and generous help on my job search and academic career development.

I would like to thank Prof. Nelson Sepúlveda, Prof. Hassan Khalil, and Prof. Ranjan Mukherjee for kindly serving on my academic committee and offering me insightful suggestions on my research. In particular, I thank Prof. Nelson Sepúlveda, who led me into the area of vanadium dioxide. His group have contributed significantly on the experimental side of my research projects including fabrication, experimental setup, and data acquisition.

I thank Prof. Guoming Zhu for his robust control course, which helped me to accomplish the work in Chapter 8. I also benefited a lot from Prof. Mark Iwen's course on compressive sensing, and the course on signal compression by Prof. Hayder Radha. These two courses have been helpful for the work in Chapter 2, Chapter 3, and Chapter 4.

I am grateful to my colleagues at MSU for their help and discussions: Dr. Emmanuelle Merced, David Torres, Dr. Hong Lei, Dr. Feitian Zhang, Dr. Jianxun Wang, Dr. Ahmad T. Abdulsadda, Dr. Alex Esbrook, Dr. Jianguo Zhao, Sanaz Behbahani, Jason Greenberg, Ali Abul, Tongyu Wang, Bo Song and many others. Special thanks are due to Emmanuelle and David, for their consistent support on the experimental side of my research projects. I would like to thank friends at MSU who offered me generous help: Dr. Fang Hou, Dr. Jiying Li, Miao Yu, Dr. Baolin Yu, Dr. Zheng Fan and Xiaofeng Zhao.

I also want to thank all of the administrators and staff members in the ECE department for their assistance during my study and life here.

I am grateful for the financial support for my research by the National Science Foundation (CMMI 0824830, CMMI 1301243, ECCS 0547131).

Most of all, I am foremost thankful for my family. I would like to thank my mother, Lizhen Zhang, my father, Yunting Zhang, and my brother, Jie Zhang for their everlasting support in all my endeavors. I am deeply indebted to my dear wife, Ran Duan, for her constant love, support and encouragement.

TABLE OF CONTENTS

LIST OF TABLES	x
LIST OF FIGURES	xi
Chapter 1 Introduction	1
1.1 Modeling, Identification, and Control of Hysteretic Systems	1
1.1.1 Modeling	1
1.1.2 Identification	2
1.1.3 Control	3
1.2 Modeling and Control of Vanadium Dioxide (VO_2) Microactuators	4
1.2.1 Modeling and Inverse Compensation	5
1.2.2 Self-sensing Feedback Control	6
1.2.3 Robust Control	7
1.3 Contribution and Organization	8
1.3.1 Overview of Contribution	8
1.3.2 Organization	10
Chapter 2 Kullback-Leibler (KL) Divergence-based Optimal Compression of the Preisach Operator	11
2.1 Problem Formulation	11
2.2 Information Loss Metric: KL Divergence-based Measure	12
2.3 Optimal Compression Scheme	14
2.4 Experimental Results	16
2.4.1 Experimental Characterization	16
2.4.2 Compression Performance	17
Chapter 3 Entropy-based Optimal Compression of the Generalized Prandtl-Ishlinskii (GPI) Model	22
3.1 Problem Formulation	23
3.2 Optimal Compression Scheme	25
3.3 Information Loss Metrics: Entropy-based Measure	27
3.4 Scaling of the Weights for the GPI Model	29
3.5 Simulation Results	30
3.5.1 Case 1: Uniform Distribution for the Scaled Weights	32
3.5.2 Case 2: Scaled Weights with One Prominent Peak	33
3.5.3 Case 3: Scaled Weights with Two Prominent Peaks	35
3.5.4 Case 4: Random Distribution for the Scaled Weights	35
3.5.5 Computational Time for the Algorithms	37
3.5.6 Comparison with a Traditional Model Identification Approach	38

3.6	Experimental Results	39
3.6.1	Compression Performance	42
3.6.2	Model Verification	43
Chapter 4	Compressive Sensing-based Preisach Operator Identification	46
4.1	Problem Formulation	46
4.2	Compressive Sensing Scheme for Identifying the Preisach Operator	50
4.2.1	Overview of Compressive Sensing	50
4.2.2	Compressive Sensing for the Preisach Operator	52
4.3	Simulation Results	57
4.4	Experimental Results	63
4.4.1	Measurement Setup	63
4.4.2	Identification and Verification	64
Chapter 5	Modeling and Inverse Compensation of Non-monotonic Hysteresis based on the Preisach Operator	70
5.1	Experimental Characterization of VO ₂ -coated Microactuators	71
5.1.1	Material Preparation and Experimental Setup	71
5.1.2	Characterization of Non-monotonic Hysteresis	73
5.2	Non-monotonic Hysteresis Model	76
5.2.1	Actuation Effect due to Phase Transition	76
5.2.2	Differential Thermal Expansion Effect	77
5.3	Model Identification and Validation	79
5.3.1	Parameter Identification	79
5.3.2	Experimental Results	80
5.4	Inverse Compensation	84
5.4.1	Inverse Compensation Algorithm	89
5.4.2	Experimental Validation	92
Chapter 6	Modeling and Inverse Compensation of Hysteresis using an Extended GPI (EGPI) model	94
6.1	EGPI model for Non-monotonic Hysteresis	94
6.2	Inverse Compensation Algorithm	95
6.3	Experimental Results: Modeling	98
6.3.1	Curvature-temperature Hysteresis of a VO ₂ -coated Microactuator	99
6.3.2	Resistance-temperature Hysteresis of a VO ₂ Film	102
6.4	Inverse Compensation Results	104
6.4.1	Simulation	104
6.4.2	Experimental Verification	106
Chapter 7	A Composite Hysteresis Model in Self-Sensing Feedback Control of VO₂-integrated Microactuators	109
7.1	Experimental Procedures	110
7.1.1	VO ₂ -integrated Actuator Fabrication	110
7.1.2	Experimental Setup	111

7.1.3	Measurement of Hysteretic Behavior	113
7.2	Proposed Composite Model for Self-sensing	115
7.2.1	Main Idea	115
7.2.2	Temperature Surrogate $g(T)$ based on a GPI Model	119
7.2.3	Estimated Deflection $\hat{D}$ Based on an EGPI Model	120
7.3	Model Identification and Verification	121
7.3.1	Model Identification	121
7.3.2	Model Verification	127
7.4	Self-sensing-based Feedback Control	129
7.4.1	Step Reference Tracking	129
7.4.2	Sinusoidal Reference Tracking	130
7.4.3	Multi-frequency Reference Tracking	130
Chapter 8	Robust Control of VO₂ Microactuators using Self-Sensing Feedback . .	133
8.1	Experimental Procedures	133
8.1.1	VO ₂ Deposition	133
8.1.2	Measurement Setup	134
8.2	Self-Sensing Deflection	135
8.3	Robust Controller Design	137
8.3.1	Modeling of VO ₂ Microactuator	138
8.3.2	Robust Controller Design	141
8.4	Experimental Result	145
8.4.1	Step Reference Tracking	146
8.4.2	Multi-frequency Signals Reference Tracking	148
8.4.3	Noise Rejection for Multi-frequency Signals Reference Tracking	150
Chapter 9	Conclusions and Future Work	153
9.1	Conclusions	153
9.2	Future Work	154
APPENDICES		156
	Appendix A Review of the Preisach Operator	157
	Appendix B Review of the Prandtl-Ishlinskii Models	161
BIBLIOGRAPHY		166

LIST OF TABLES

Table 3.1	Parameters of the GPI model envelope functions.	31
Table 3.2	Compression performance comparison: the uniform case.	32
Table 3.3	Compression performance comparison: the case of one peak.	35
Table 3.4	Compression performance comparison: the case of two peaks.	36
Table 3.5	Compression performance comparison: the case of random distribution. . .	36
Table 3.6	Identified parameters of the envelope functions.	39
Table 3.7	Modeling verification error comparison.	43
Table 6.1	Parameters of the GPI model and the EGPI model for hysteresis of a VO ₂ -coated microactuator.	100
Table 6.2	Parameters of the GPI model and the EGPI model for hysteresis of a VO ₂ film.	102
Table 6.3	Parameter of the EGPI model	106
Table 7.1	Identified parameters of the GPI model.	122
Table 7.2	Identified parameters of the EGPI model.	123
Table 8.1	Steady-state values of step experiments for system identification.	140
Table 8.2	Controller comparison for step reference tracking.	148
Table 8.3	Controller comparison for multisinusoidal reference tracking.	150
Table 8.4	Controller comparison for multisinusoidal reference tracking with noise. .	151

LIST OF FIGURES

Figure 1.1	Superimposed SEM pictures of the $300\text{ }\mu\text{m}$ VO_2 -coated silicon cantilever taken when the substrate temperature was $30\text{ }^\circ\text{C}$ (lower curvature) and $90\text{ }^\circ\text{C}$ (higher curvature), respectively.	5
Figure 1.2	Non-monotonic hysteresis behavior in a VO_2 microactuator. The good repeatability of the actuation behavior is also shown.	6
Figure 2.1	Experimental setup for resistance vs temperature measurements of a VO_2 film.	16
Figure 2.2	(a) Measured $-\log_{10}(R)-T$ hysteresis in VO_2 . (b) Identified Preisach density function.	17
Figure 2.3	Uniform discretization.	18
Figure 2.4	Non-uniform discretization: (a) Using maximum of information losses J_∞ as cost function. (b) Using sum of information losses J_1 as cost function. (c) Using KL divergence J_{KL} of information losses as cost function. . . .	19
Figure 2.5	(a) Input temperature sequence for model validation. (b) Output error comparison between uniform discretization, maximum of information loss, sum of information loss and KL divergence of information loss. . . .	20
Figure 2.6	(a): Desired output of sinusoidal shape sequence. (b): Inverse compensation errors between uniform discretization, max of information loss, sum of information loss and KL divergence of information loss.	21
Figure 3.1	Schematic illustrating the compression of a weighting function. The solid-line segments are the original weighting function, and the dotted-line segments are the new weighting function.	24
Figure 3.2	Illustration of the radius-dependent output range for a generalized play operator.	30
Figure 3.3	Weighting function (uniform case) of the GPI model: (a) Unscaled. (b) Scaled.	33
Figure 3.4	(a) Input sequence. (b) Input vs output for the GPI model with uniform weight function.	34

Figure 3.5	Scaled weighting function (one-peak case).	34
Figure 3.6	Scaled weighting function (two-peak case).	35
Figure 3.7	Scaled weighting function (the random case).	36
Figure 3.8	Comparison of average optimization time. Note the log scale.	37
Figure 3.9	Comparison of the number of information loss evaluations. Note the log scale.	38
Figure 3.10	(a) A third-order reversal input sequence. (b) The corresponding output sequence. (c) The output prediction error between the Entropy Sum approach and output optimization approach.	40
Figure 3.11	(a) A random input sequence, and (b) the corresponding output prediction error performance. (c) The output prediction performance based on 50 random input sequences.	41
Figure 3.12	The performance of a GPI model (30 plays) in modeling of the resistance-temperature hysteresis in VO_2	42
Figure 3.13	(a) Identified weights for all the play operators of the GPI model. (b) The scaled weights for the GPI operators.	42
Figure 3.14	Parameters of the compressed GPI model: uniform compression.	43
Figure 3.15	Parameters of the compressed GPI model: (a). Entropy Sum. (b) Entropy Max.	44
Figure 3.16	(a) A new temperature input sequence for model verification. (b) Corresponding output sequence. (c) The output prediction error comparison of Entropy Sum Unscaled approach and the Entropy Sum Scaled approach.	45
Figure 4.1	Illustration of a discretization of the Preisach density function, where the discretization level $L = 4$	47
Figure 4.2	The “damped oscillation” sequence for Preisach operator identification ($L = 30$).	49
Figure 4.3	(a) Signal q showing the sparseness; (b) signal q_{order} is more approximately sparse than q ; (c) the reconstruction performance comparison based on “CS” and “CS Order”.	59

Figure 4.4	(a) Density w reconstruction error comparison; (b) the modeling error comparison.	60
Figure 4.5	(a) Density reconstruction error with varying measurement noise based on “CS”; (b) the average identification run-time comparison.	60
Figure 4.6	(a) A random input sequence for model validation; (b) corresponding output under the random input sequence in (a); (c) model estimation error comparison.	62
Figure 4.7	a) Side view schematic for the measurements setup for deflection of a microactuator with an integrated heater; b) Top view of the VO ₂ -based integrated actuator devices.	64
Figure 4.8	(a) Input-output data for identifying Preisach operator ($L = 30$); (b) true density function identified based on 467 measurements.	66
Figure 4.9	Density reconstruction error comparison based on (a) Least Squares; (b) CS; (c) CS Order; (d) CS Order Non-negative approaches.	67
Figure 4.10	The average RMSE modeling error comparison.	68
Figure 4.11	Density reconstruction error based on (a) a random input sequence for model validation; (b) output of the random input sequence in (a); (c) model estimation errors.	69
Figure 5.1	Setup used for measuring the cantilever tip deflection as a function of temperature.	72
Figure 5.2	Illustration of the geometric relationship between the curvature and the tip deflection of a bent cantilever.	73
Figure 5.3	Deflection as a function of time through heating and cooling temperature steps. There is no observable creep.	74
Figure 5.4	Proposed model. (a): Measured non-monotonic curvature-temperature hysteresis and that based on the proposed model. (b): Modeling error for the entire temperature sequence.	81
Figure 5.5	Identified Preisach density function for the proposed model.	82
Figure 5.6	Modeling error with the signed Preisach operator for the entire temperature sequence.	83

Figure 5.7	The negative of the identified density values for the signed Preisach operator. The negative is taken here so that the negative elements of the density function can be seen (on top); the positive elements are now flipped to the bottom of the plane, which are not visible here.	83
Figure 5.8	Modeling error with a polynomial model for the entire temperature sequence.	83
Figure 5.9	(a): A randomly chosen temperature input sequence for model validation. (b): Errors in predictions by different models under the input sequence. . .	84
Figure 5.10	Illustration of the variables $d_1^{(k)}$ and $d_2^{(k)}$ used in inversion.	87
Figure 5.11	(a): Open-loop inverse control performance for the proposed model and the polynomial model. (b): Inverse compensation errors.	93
Figure 6.1	A GPI model and an EGPI model with identical weights of generalized play operators.	95
Figure 6.2	The performance of modeling curvature-temperature hysteresis of a VO ₂ -coated microcantilever based on: (a) GPI model. (b) EGPI model.	101
Figure 6.3	The performance of modeling the resistance-temperature hysteresis of a VO ₂ film based on: (a) GPI model. (b) EGPI model.	103
Figure 6.4	Model verification of the resistance-temperature hysteresis in a VO ₂ film: (a) a random temperature sequence. (b) corresponding resistance output. (c) Modeling comparison between the GPI model and EGPI model.	105
Figure 6.5	Simulation verification of the inverse algorithm. Hysteresis relationship: (a) Input sequence. (b) Input-output of the EGPI model.	107
Figure 6.6	Compensation of hysteresis in simulation: (a) Input-output of the inverse EGPI model. (b) The relationship of the desired output and the actual output after hysteresis compensation.	107
Figure 6.7	(a). Inverse compensation performance in experiment. (b). Inversion compensation error for the EGPI model.	108

Figure 7.1	Fabrication process flow for the VO ₂ -integrated actuator. a) Deposition of SiO ₂ (1 μ m) by PECVD; (b) deposition of VO ₂ (270 nm) by PLD; (c) patterning (etch) of VO ₂ by RIE; (d) deposition of SiO ₂ (0.4 μ m) by PECVD; (e) patterning (etch) of SiO ₂ by RIE; (f-g) deposition of Ti/Au by evaporation and patterning by lift-off; (h) RIE of SiO ₂ for device pattern; (j) cantilever released by XeF ₂ isotropic etching of Si.	112
Figure 7.2	The VO ₂ -integrated microactuator used in this work, with length 425 μ m and width 65 μ m.	113
Figure 7.3	(a) The hysteresis between the resistance and the current; (b) the hysteresis between the deflection and the current.	114
Figure 7.4	(a) The hysteresis between the deflection and the resistance; (b) the resistance sequence; (c) zoom-in plot of the hysteresis between the deflection and the resistance, revealing a non-nested structure.	116
Figure 7.5	The hysteresis between the deflection and the current under varying input frequencies; (b) The hysteresis between the deflection and the resistance under varying input frequencies.	117
Figure 7.6	(a) The comparison between the GPI model prediction and experimental measurement for the asymmetric hysteresis between the resistance output and the current input; (b) the comparison between the EGPI model prediction and experimental measurement for the non-monotonic hysteresis between the deflection output and the current input.	123
Figure 7.7	(a) Performance of the self-sensing scheme using the composite model; (b) the self-sensing error based on the composite model.	124
Figure 7.8	Performances of the self-sensing schemes using (a) a Preisach model; (b) an EGPI model; (c) a high-order polynomial model.	125
Figure 7.9	Model accuracy and the running time comparison between the composite model, the Preisach model, the EGPI model, and the polynomial model.	127
Figure 7.10	(a) A randomly chosen current input sequence for self-sensing model verification; (b) the experimental deflection measurement under the random current input sequence; (c) errors in predictions by different self-sensing approaches.	128
Figure 7.11	Block diagram of the closed-loop control system with self-sensing.	129
Figure 7.12	Experimental performance of tracking a step reference under different self-sensing schemes.	130

Figure 7.13	(a) A sinusoidal reference signal for tracking control of the VO ₂ -integrated microactuator; (b) experimental tracking errors under different self-sensing schemes.	131
Figure 7.14	(a) A multi-frequency reference signal for tracking control of the VO ₂ -integrated microactuator; (b) experimental tracking errors under different self-sensing schemes.	132
Figure 8.1	(a) VO ₂ film resistance, and (b) VO ₂ -coated microactuator actual deflection as a function of temperature through a heating-cooling cycle (20-85 °C). Both variables were simultaneously measured.	136
Figure 8.2	(a) VO ₂ -coated microactuator actual deflection as a function of VO ₂ film resistance during the heating-cooling cycle. A polynomial function of degree 9 was used to model the deflection-resistance mapping. (b) Maximum model error obtained from the major heating and cooling curves. . .	137
Figure 8.3	Block diagrams of the (a) simplified physical closed-loop control system with self-sensing and (b) closed-loop system augmented with weighted functions.	138
Figure 8.4	(a) Framework of H_{∞} control for the system and (b) robust performance test by augmenting the uncertainty Δ to M.	142
Figure 8.5	(a) Actual, and (b) self-sensed microactuator deflection under self-sensed, closed-loop PID and robust control through a series of step reference inputs.	147
Figure 8.6	(a) Actual deflection error and (b) controller effort for the PID and robust control approaches through the step reference tracking experiment. . . .	147
Figure 8.7	Microactuator deflection response to a multisinusoidal reference input under PID and robust control.	149
Figure 8.8	(a) Actual deflection error and (b) controller effort for the PID and robust control approaches in the multisinusoidal reference tracking experiment. .	149
Figure 8.9	(a) Microactuator deflection response and (b) error under a multisinusoidal reference input under PID and robust control with induced white noise in current input to the system.	151
Figure 8.10	(a) Control effort and (b) Peltier input for a multisinusoidal reference input under PID and robust closed-control with induced white noise in current input to the system.	152

Figure 8.11	Frequency spectrum analysis of the tracking errors under the robust controller and the PID controller, for scenarios with and without injected actuation noise.	152
Figure A.1	Illustration of uniform discretization of the Preisach plane, where the discretization level $M = 4$	159
Figure A.2	Inverse compensation of hysteresis.	160
Figure B.1	Input-output relationships of (a) a classical play operator with radius r ; (b) a generalized play operator with radius r (shown as solid curves). . . .	163

Chapter 1

Introduction

In this chapter, a brief background on modeling, identification, and control of hysteretic systems is presented. Limitations of the existing work on modeling and identification of hysteretic systems are discussed. Afterwards, the motivations of inverse compensation, self-sensing feedback control, and robust control for vanadium dioxide (VO_2) microactuators are briefly discussed. At last, an overview of the contributions and the organization of the dissertation are presented.

1.1 Modeling, Identification, and Control of Hysteretic Systems

1.1.1 Modeling

The term “hysteresis” was coined by James A. Ewing in his 1881 study of ferromagnetism [1]. Hysteresis is a nonlinear effect that occurs in a wide range of areas, such as biology [2], economics [3], ferromagnetic materials [4] and various smart materials [5–8]. There has been extensive work dealing with modeling and control of systems with hysteresis. Hysteresis models can be roughly classified as physics-based and phenomenology-based. Jiles and Atherton [4] proposed a physics-based hysteresis model for ferromagnetics. While physics-based model may be valid to a limited quantity of systems, phenomenology-based models, such as the Preisach operator [9–12], generalized Prandtl-Ishlinskii (GPI) model [13–16], Duhem model [17], Bouc-Wen model [18], and Maxwell model [19], are often applicable to a broader class of systems with hys-

teresis, and thus have been adopted more extensively to capture hysteresis nonlinearity. Among them, the Preisach operator and the GPI model are widely adopted and have been proven effective in capturing different forms of hysteresis.

The practical utilization of the Preisach operator mostly involves uniform discretization of the density function on the Preisach plane [9, 10, 12]. For example, Tan and Baras proposed to approximate the Preisach density function with a piecewise constant function, where the Preisach density function is discretized into a grid consisting of equal-sized cells [9]. It is anticipated and verified that the accuracy of the model improves as the number of discretization level increases [12]; however, computational complexity and data storage cost also increase with the number of discretization level, posing challenges in parameter identification and control of systems with hysteresis. Similarly, existing work on the GPI model has typically adopted some predefined play radii [13, 20–22], the modeling performance of which could be far from optimal. While it is generally true that the modeling performance improves with an increasing number of play operators, similarly, computational and data storage costs will also increase for the model and the corresponding model-based inverse compensation. Obtaining accurate hysteresis models while maintaining a relatively low calculation and storage costs is thus an issue of practical interest.

1.1.2 Identification

The Preisach operator consists of weighted superposition of hysterons. Parameter identification based on the Preisach operator usually involves discretization of the Preisach density function in one form or another, and one effective method is to approximate the density function with a piecewise constant function [9]. Both online [12, 23] and offline [9, 24–26] schemes can be adopted for model identification. When the discretization level is L , there are $L(L + 1)/2$ cells with different density values [27]. The input needs to provide sufficient excitation for all the density

values for model identification [12]. One example of such inputs takes the form of damped oscillations, which produces nested hysteresis loops [12]. The input sequence should contain at least $L(L+1)/2$ elements to identify all the densities. When the discretization level is chosen larger, the corresponding Preisach operator could better capture the actual hysteresis, but the identification would require a larger number of measurements. For instance, in [24], the Preisach density function was discretized into 200 levels, and at least 20,100 measurements would need to be taken and processed to identify all the density values. In [26], a 20-level Preisach operator was adopted to characterize the displacement-temperature hysteresis of a VO_2 -coated microactuator. In order to capture the hysteresis under quasi-static condition, a relatively long wait time was needed for each measurement due to the slow thermal dynamics, resulting in long experiment time for collecting the required data for model identification [26]. Therefore, it is of great interest to design a more efficient identification approach that requires less input-output data.

1.1.3 Control

With the fast development of smart material-actuated hysteretic systems, there has been an increasing amount of work in control schemes. Among them, an important class is inverse compensation. When experiments are operate in quasi-static condition [6], the inverse compensation problem is simplified to: given a desired output value, calculate an input sequence such that the final value of the plant reaches the desired value. So the hysteresis effect is approximately cancelled out in this manner. Inversions of a few phenomenological models have been reported [9, 18, 20, 22, 27, 28]. The inversion of the Preisach model is typically derived based on numerical iteration [27], analytical inversion of the GPI model can often be derived analytically [20, 28], which facilitates the real-time control implementation. Note that analytical inversion of a GPI model requires that all the generalized play operators have the same envelope functions, limiting its ability in modeling

complex hysteresis.

Although inverse compensation is effective, it is also highly computationally demanding and does not perform robustly against disturbances. In order to control systems with hysteresis, various feedback control approaches have been proposed [9, 12, 29, 30]. Robust control theory has been used in systems to reduce environmental disturbances and plant uncertainties, but such studies have been typically limited to conventional smart materials, such as piezoelectric-based actuators [31–34], where the controllers were designed to control the deflection of piezoelectric microactuators based on charge measurements.

1.2 Modeling and Control of Vanadium Dioxide (VO_2) Microactuators

VO_2 is a novel smart material that undergoes a thermally induced solid-to-solid phase transition around 68 °C [35]. During the transition, the material's crystalline structure changes from a monoclinic phase (M_1) at low temperatures to a tetragonal phase (R) at high temperatures, which results in drastic changes in multiple physical properties (including resistance [8], induced mechanical stress [36], and optical transmittance [37]) and pronounced hysteresis with respect to temperature. These characteristics make VO_2 a promising multifunctional material for sensors [38], actuators [36, 39, 40], and memory applications [41]. The actuation potential of VO_2 was not noticed until recently [36]. As shown in Fig. 1.1, by coating VO_2 on a microstructure (e.g., a silicon cantilever), thermally actuated micro-benders can be created, which have shown full reversible actuation, large bending, and high energy density [40], making them particularly suitable for applications such as micromanipulation and microrobotics.

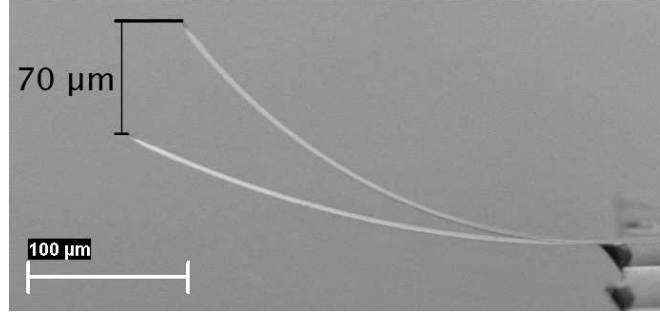


Figure 1.1: Superimposed SEM pictures of the 300 μm VO_2 -coated silicon cantilever taken when the substrate temperature was 30 $^\circ\text{C}$ (lower curvature) and 90 $^\circ\text{C}$ (higher curvature), respectively.

1.2.1 Modeling and Inverse Compensation

The realization of the potential of VO_2 -coated microactuators, however, is greatly hindered by their sophisticated non-monotonic hysteretic behavior (shown in Fig. 1.2) resulting from two competing actuation effects. The first actuation effect is due to the internal stress generated during the phase transition, which is inherently hysteretic with respect to temperature. The second actuation effect is due to the differential thermal expansion of the VO_2 layer and the substrate, which causes an opposite bending effect. While the thermal expansion effect persists throughout the temperature range, the stress generated during the VO_2 's structural changes dominates across the phase transition [39]. As a result, the relationship between the bending curvature and temperature is *non-monotonic* when the temperature is raised or lowered monotonically. It is crucial to capture the non-monotonic hysteresis behavior in VO_2 microactuators.

Most studies on modeling and inverse compensation of hysteresis in smart materials have focused on monotonic hysteresis nonlinearities [9, 10, 28, 42–47], where a monotonic input causes a monotonic output. A special type of non-monotonic hysteresis with butterfly-shaped hysteresis loops was investigated by Drincic *et al.* [48]; however, the study there was focused on hysteresis loops that can be converted to monotonic hysteresis through uni-modal mappings. For the model consisting of a classical Prandtl-Ishlinskii (CPI) model and a memoryless function, the authors

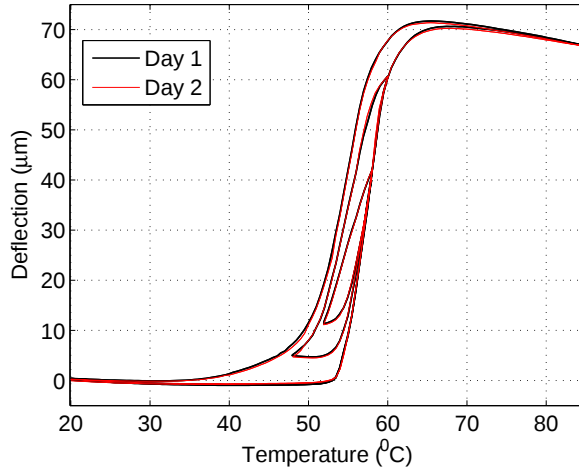


Figure 1.2: Non-monotonic hysteresis behavior in a VO₂ microactuator. The good repeatability of the actuation behavior is also shown.

proposed an iterative scheme for its inversion [49], but the convergence of the inverse algorithm was not considered. Inverse compensation of non-monotonic hysteresis needs to be developed for VO₂ microactuators.

1.2.2 Self-sensing Feedback Control

For the control of micro devices, external sensing systems, such as laser scattering [36] and interferometry [50], are often undesirable or even infeasible due to their sizes and complexity, and self-sensing provides a cost-effective alternative. In self-sensing, the variable of interest (often a mechanical signal) is estimated based on another variable (typically an electrical signal) that is much easier to obtain. Existing work on self-sensing of actuators has mainly involved traditional smart materials, such as piezoelectrics [29, 51, 52], shape memory alloys (SMAs) [30, 53, 54], and magnetorheological fluids [55]. For example, Ivan *et al.* implemented self-sensing for piezoelectric actuators, where both the displacement and the external force at the tip of the cantilever were estimated based on the current measurement, and a Prandtl-Ishlinskii model was adopted to compensate for the remaining hysteresis nonlinearity [52]. In [53], the strain feedback of the

SMA-actuated flexures for motion control was estimated from resistance measurement using a high-order polynomial model. Polynomial models were also utilized to estimate the strain or gripper motion of the SMA-based grippers [30, 54]. Although the hysteresis gap between strain and resistance can be decreased by changing the pretension force, the remaining hysteresis still poses challenges for precision control of these grippers.

The self-sensing of VO₂-based microactuators presents new challenges. In particular, the resistance change is due to an insulator-to-metal transition while the mechanical change is due to a structural phase transition [35]. Although strongly correlated, these two different phase transitions do not occur simultaneously and thus the relationship between deflection and resistance is hysteretic and highly complicated. In [56], a memoryless Boltzmann function was utilized for self-sensing and a proportional-integral controller was implemented based on the self-sensing signal. However, memoryless functions-based self-sensing schemes cannot capture the inherent deflection-resistance hysteresis and result in large sensing errors, which poses a significant limitation to tracking control accuracy. A novel composite self-sensing model and proportional - integral control based on the composite self-sensing will be studied.

1.2.3 Robust Control

Robust control using self-sensing feedback needs to be designed to improve the feedback control robustness for VO₂ microactuators. Although external disturbances and model uncertainties were considered for the controller designs, the error between the actual deflection and the reference error was not addressed explicitly. The robust controllers in [33,34] were synthesized for suppression of piezoelectric structure vibrations by self-sensing the rate of strain change, where tracking desired reference signals was not a concern. The work done in [33] followed a similar control framework as in [31], but it was designed to follow a desired deflection value of zero (in order to reduce vi-

brations). Although the controller design in [34] accommodates constraints on control effort, it does not account for effects of model uncertainties, hysteresis, or disturbances. In terms of other hysteretic materials, such as SMAs, there has been no reported work in robust control using self-sensing for deflection control. A robust controller will be developed, which takes into account the error in modeling temperature-deflection hysteresis, and environmental disturbances, in order to minimize the tracking error. Unlike previous work on robust position control of hysteretic microactuators based on self-sensing [31–34], the controller in this work takes into consideration the error between the desired and actual deflection values in order to precisely control the microactuator. The performance of the robust controller is also compared to a proportional-integral-derivative (PID) controller.

1.3 Contribution and Organization

1.3.1 Overview of Contribution

First, tools from information theory, namely KL divergence and entropy, are utilized to optimally compress the Preisach operator and the GPI model under given complexity constraints. The compressed hysteresis models are more accurate while maintaining relatively low calculation and storage complexity. While due to the particular setting of the Preisach plane, the optimal compression of the Preisach operator involves an exhaustive search, the optimal compression of the GPI model is reformulated as an optimal control problem and solved with dynamic programming. The proposed schemes are verified in simulation and experimental results involving the hysteresis between the resistance and the temperature of a VO_2 film.

Second, identification of the Preisach operator is studied under the compressive sensing framework that requires much fewer measurements. The proposed approach adopts the discrete cosine

transform (DCT) transform of the output data to obtain a sparse vector. Sparser vector is further obtained assuming the order of all the output data are known. The model parameters can be efficiently reconstructed using the proposed scheme. The least-squares scheme is also realized, and is compared with the proposed approach using the same number of measurements. Root-mean-square error (RMSE) is adopted to examine the identified model parameters and model estimation performances. The proposed identification approach is shown to have better identification performance than the least-squares scheme through both simulation and experiments involving a VO₂-integrated microactuator.

Third, physics-motivated non-monotonic hysteresis models that account for the two competing actuation mechanisms are presented. The first mechanism is the stress resulting from structural changes in VO₂, which is modeled with a monotonic Preisach operator or a GPI model. The second mechanism is the differential thermal expansion effect, which is modeled with a memoryless operator. Efficient inverse compensation schemes are developed for the proposed non-monotonic hysteresis models. For the non-monotonic model based on the Preisach operator, the inversion complexity is studied; for the non-monotonic model based on the GPI model, the inversion is developed based on fixed-point iteration with which the convergence conditions of the algorithm are derived. The proposed modeling and compensation schemes are validated experimentally.

Fourth, self-sensing feedback control for VO₂ microactuators is studied. The proposed composite self-sensing approach exploits the physical understanding that both the resistance and the deflection have different hysteretic relationships with the temperature. The steady state current is used as a surrogate for the temperature of VO₂. The self-sensing model is obtained by cascading an extended GPI (EGPI) model with the inverse of a GPI model. The performance of the self-sensing scheme is evaluated experimentally with proportional-integral control.

Finally, an H_∞ robust controller is further designed and implemented for precision deflection

control. A ninth-order polynomial is adopted to model the self-sensing relationship between the deflection and the resistance. The uncertainties produced by the hysteresis between the deflection and the temperature input and the error in the self-sensing model are accommodated by the proposed controller. The robust controller is demonstrated through step and multisinusoidal reference tracking experiments with simulated white noise current signal.

1.3.2 Organization

In Chapter 2, the optimal compression of the Preisach operator is presented. In Chapter 3, we present the optimal compression of the GPI model. The compressive sensing-based Preisach operator identification is proposed in Chapter 4. In Chapter 5, we discuss the non-monotonic hysteresis modeling and inverse compensation based on the Preisach operator. The EGPI model and its inversion are studied in Chapter 6. In Chapter 7, a composite hysteresis model is proposed for self-sensing feedback control of VO_2 microactuators. Robust control for VO_2 microactuators is presented in Chapter 8. Conclusions and future work are provided in Chapter 9.

Chapter 2

Kullback-Leibler (KL) Divergence-based Optimal Compression of the Preisach Operator

In this chapter, a novel scheme to optimally compress the Preisach operator is proposed. The KL divergence is utilized to quantify the information loss in approximating the Preisach density function as piecewise-constant functions. In particular, the proposed cost function incorporates both the largest cell information loss and the total information loss, for a given discretization scheme on the Preisach plane. Exhaustive search is conducted to find the optimal discretization scheme. The proposed approach is applied to the modeling of the hysteretic relationship between resistance and temperature of a VO_2 film, and its effectiveness is further examined in open-loop inverse compensation experiments. The proposed discretization scheme is compared with two other approaches and with uniform discretization, and the effectiveness of the proposed approach is validated in both model verification and inverse compensation.

2.1 Problem Formulation

KL divergence, or relative entropy, characterizes the distance between probability distribution functions [57]. KL divergence has been used extensively in statistics [58], pattern recognition [59],

and signal processing [60]. To motivate this approach in discretization of the Preisach operator, consider the problem of approximating a probability distribution function (pdf) $f_p(x)$ in a certain region with a uniform pdf $f_q(x)$. If $f_p(x)$ is a uniform distribution, then it can be approximated without error by $f_q(x)$. However, much of its information will be lost if $f_p(x)$ varies greatly from point to point.

For the Preisach operator, the number of discretization levels has a direct impact on the complexity in model representation, identification, and inverse compensation, and is thus taken as the complexity measure. The choice of the Preisach operator discretization is a critical issue that determines the accuracy in approximating the Preisach operator and the complexity in implementing the inverse Preisach operator. Consider the problem of approximating some arbitrary Preisach density function with a cell-wise constant function. For a given discretization level (thus a given level of algorithmic complexity), it is desirable to discretize the density function in such a way that the original density function has the least information loss approximating it using a constant within each cell. In other words, coarser (finer, resp.) discretization should be applied in regions with smaller (larger, resp.) density variation so as to minimize the overall information loss in the approximation process.

Our problem is thus formulated as: given the number of discretization level, find the optimal discretization scheme that minimizes the information loss in representing the original Preisach operator by the approximating one.

2.2 Information Loss Metric: KL Divergence-based Measure

For continuous random variables, the KL divergence is defined as:

$$D_{LK}(P||Q) = \int_X f_p(x) \log \frac{f_p(x)}{f_q(x)} dx, \quad (2.1)$$

where f_p and f_q represent the pdfs of P and Q , respectively. We use the convention that $0 \log \frac{0}{q} = 0$, $p \log \frac{p}{0} = \infty$. It can be shown that the KL divergence between two pdfs is always nonnegative, and is zero if and only if the two probability distributions are identical [57].

The concept of KL divergence can be applied to Preisach density function discretization in the following way: assume that a discretized density function with high fidelity is known (knowing the true infinite-dimensional density function is not practical). The original density can be approximated with a piecewise constant function compatible with a discretization grid using a lower discretization level; namely, the approximating density takes a constant value within each discretization cell, but the value varies from cell to cell [9]. The KL divergence between the (normalized) original density restricted to a cell and the uniform distribution can then capture the information loss in that cell.

In particular, for a certain discretization, the pdf $p_{i,j}$ within each cell is first calculated and the amount of information loss $H_{i,j}$ for representing $p_{i,j}$ with a uniform density $q_{i,j}$ is computed. $T_{i,j}$ is defined as the integral of μ over cell (i, j) :

$$T_{i,j} = \int \int_{Cell(i,j)} \mu(\beta, \alpha) d\beta d\alpha. \quad (2.2)$$

Then the probability density functions $p_{i,j}$ and $q_{i,j}$, over cell (i, j) , are defined as:

$$p_{i,j}(\beta, \alpha) = \frac{\mu(\beta, \alpha)}{T_{i,j}}, \quad (2.3)$$

$$q_{i,j}(\beta, \alpha) = \frac{1}{S_{i,j}}, \quad (2.4)$$

where $S_{i,j}$ is the total area of cell (i, j) . The KL divergence $H_{i,j}$ between $p_{i,j}$ and $q_{i,j}$ is:

$$H_{i,j} = \int \int_{cell(i,j)} p_{i,j}(\beta, \alpha) \log \frac{p_{i,j}(\beta, \alpha)}{q_{i,j}(\beta, \alpha)} d\beta d\alpha. \quad (2.5)$$

Due to the normalization process, the relative importance of each cell with respect to other cells is not captured in $H_{i,j}$. To account for this, the information loss is defined as $\mathcal{L}_{i,j}$ for cell (i, j) by weighing $H_{i,j}$ with $T_{i,j}$:

$$\mathcal{L}_{i,j} = H_{i,j} \cdot T_{i,j}.$$

The approximating Preisach density value for cell (i, j) is $T_{i,j}/S_{i,j}$.

2.3 Optimal Compression Scheme

It is of interest to investigate what is a suitable “metric” for measuring the compression error. For a given level of discretization M , the discretization variables are denoted as $\{\beta_k\}_{k=0}^M$, where $\beta_0 = v_{min}$, $\beta_M = v_{max}$ are fixed and $\beta_0 \leq \beta_1 \leq \dots \leq \beta_{M-1} \leq \beta_M$. $\mathcal{D} = \{\beta_k\}_{k=1}^{M-1}$ is called a discretization strategy. For a given $\mathcal{D}$, the weighted KL divergence $\mathcal{L}_{i,j}$ can measure how well the (normalized) density function within each cell (i, j) is approximated by a uniform distribution, but a total measure for the overall information loss is needed to evaluate the performance of the discretization strategy $\mathcal{D}$.

In searching the total measure criterion, thus achieving optimal compression, it's not desirable to have certain cells whose information loss are very large. The total information loss of all cells is another consideration. The criterion is proposed to be

$$J_{KL}(\mathcal{D}) = H \cdot D_{KL}(P_{\mathcal{L}} \| Q_u),$$

where $P_{\mathcal{L}}$ represents the (normalized) pdf of $\{\mathcal{L}_{i,j}\}$, and Q_u represents a uniform pdf with value $1/N$, where $N = M(M+1)/2$ is the total number of cells. The relative importance is similarly chosen as H and the area for every cell is assumed to be 1. This criterion takes both the *total* information loss and *maximum* information loss into consideration.

Other than uniform discretization, two other cost function approaches are considered for comparison purposes.

- $J_{\infty}(\mathcal{D}) = \max_{i,j} \mathcal{L}_{i,j}$

With J_{∞} , the largest information loss among cells is minimized, but the total information loss could be large.

- $J_1(\mathcal{D}) = \sum_{i,j} \mathcal{L}_{i,j}$

With J_1 , the total information loss is minimized, but large information losses in some regions are possible, which would result in large error when the operator works in that particular region.

Since the proposed approach considers both the maximum and total information losses, it is expected that it will best approximate the density function, compared with the other two cost function approaches and the uniform discretization scheme under the same complexity.

The compression scheme can be outlined as follows. Given an original Preisach operator, with uniform discretized density function $\mu_{i,j}, i, j = 1, \dots, L$. Discretize the density function with a lower discretization level $M < L$, and find $\mathcal{D} = \{\beta_k\}_{k=1}^{M-1}$ that minimizes $J(\mathcal{D})$. $\{\beta_k\}_{k=1}^{M-1}$ values are chosen from a discrete set. In this work the discrete set is chosen to be the input levels of the

original discretized plane: $\beta_k \in \{v_{\min} + \frac{(l-1)(v_{\max}-v_{\min})}{L} | l = 1, 2, \dots, L\}, k = 1, \dots, M-1$, and exhaustive search is conducted to find the solution that minimizes $J(\mathcal{D})$.

2.4 Experimental Results

2.4.1 Experimental Characterization

The hysteresis between resistance and temperature in a VO_2 film was used as an example to show the effectiveness of the proposed discretization scheme. A VO_2 layer was deposited by pulsed laser deposition. The film was glued with a highly thermal conductive silver paint, and in close contact with a Peltier heater. The Peltier heater was controlled with a temperature controller with 0.1 °C precision. Fig. 2.1 shows the experimental setup. The resistance of the film was measured through two electrical aluminum contacts patterned on the VO_2 film.

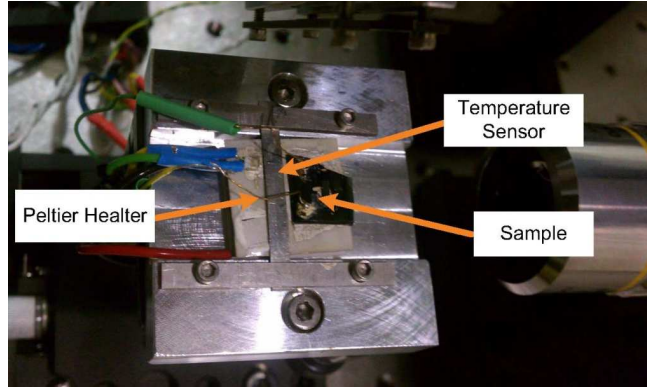


Figure 2.1: Experimental setup for resistance vs temperature measurements of a VO_2 film.

Since the measured resistance (R) changes approximately two orders of magnitude during the phase transition, $-\log_{10} R$ was taken as the output, where the negative sign is introduced so that the resulting Preisach operator has a nonnegative density function. Fig. 2.2(a) shows the measured $-\log_{10} R-T$ hysteresis loops including minor loops, and Fig. 2.2(b) shows the identified, piecewise constant density function $\mu(\beta, \alpha)$ using the data in Fig. 2.2(a), where uniform discretization on the

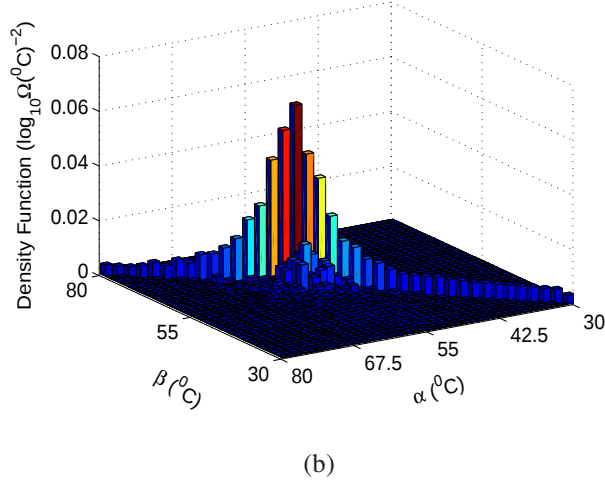
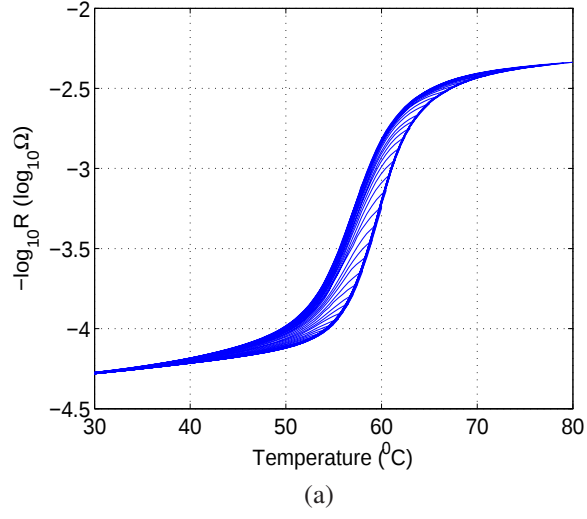


Figure 2.2: (a) Measured $-\log_{10}(R)$ – T hysteresis in VO_2 . (b) Identified Preisach density function.

Preisach density function was adopted with discretization level $M = 40$. The full range of temperature input $[v'_{\min}, v'_{\max}]$ was $[30, 80]$ °C. The offset value was identified to be $-3.308\log_{10}\Omega$, where $\log_{10}\Omega$ denotes the unit of the output.

2.4.2 Compression Performance

To conduct the compression studies, the piecewise constant Preisach density function with (uniform) discretization level 40 was taken as the “original” density function, and the approximation

to it with discretization level 4 was found. Fig. 2.3 shows the uniform discretization and Fig. 2.4 shows discretization results based on $J_\infty(\mathcal{D})$, $J_1(\mathcal{D})$, $J_{KL}(\mathcal{D})$, respectively. It can be noted from the figures that in the non-uniform discretization scheme, the discretization is finer in the center area since this is where the original density function has larger variations. However, when the cost function is chosen as $J_\infty(\mathcal{D})$ or $J_1(\mathcal{D})$, the center cells have very small area, which is actually undesirable. $J_{KL}(\mathcal{D})$ based compression comes with a better approximation of the “original” density distribution.

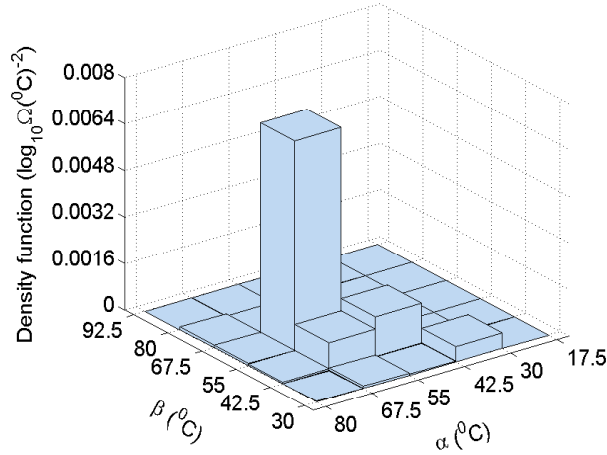
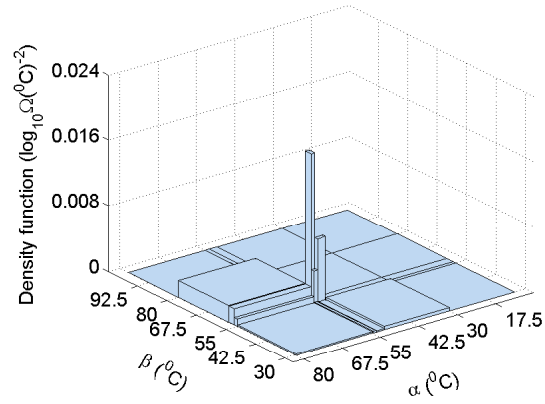
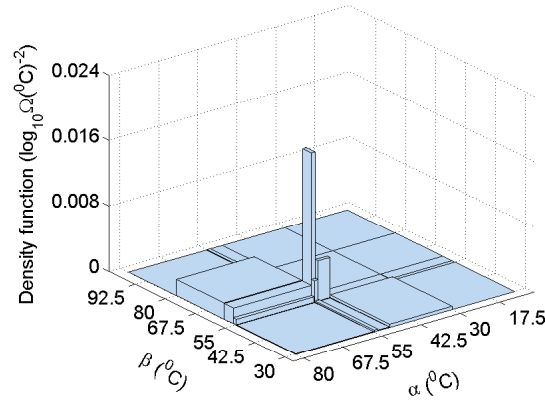


Figure 2.3: Uniform discretization.

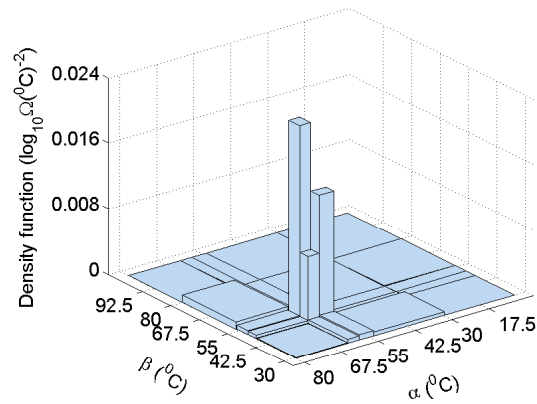
In order to examine the approximating performance of each discretization scheme, a randomly chosen input sequence for the system shown in Fig. 2.5(a) was applied. The corresponding resistance output in experiment was then obtained and compared with the predictions from the four low-complexity models mentioned above. Fig. 2.5(b) shows the prediction errors for the four models. The model based on J_{KL} was able to predict much better the resistance than the other three schemes. The largest output error using the KL scheme is below $0.23 \log_{10} \Omega$, while uniform discretization approach results in errors larger than $0.57 \log_{10} \Omega$ and other two approaches end up with larger than $1.00 \log_{10} \Omega$. The effectiveness of the proposed approach in representing the true



(a)



(b)



(c)

Figure 2.4: Non-uniform discretization: (a) Using maximum of information losses J_{∞} as cost function. (b) Using sum of information losses J_1 as cost function. (c) Using KL divergence J_{KL} of information losses as cost function.

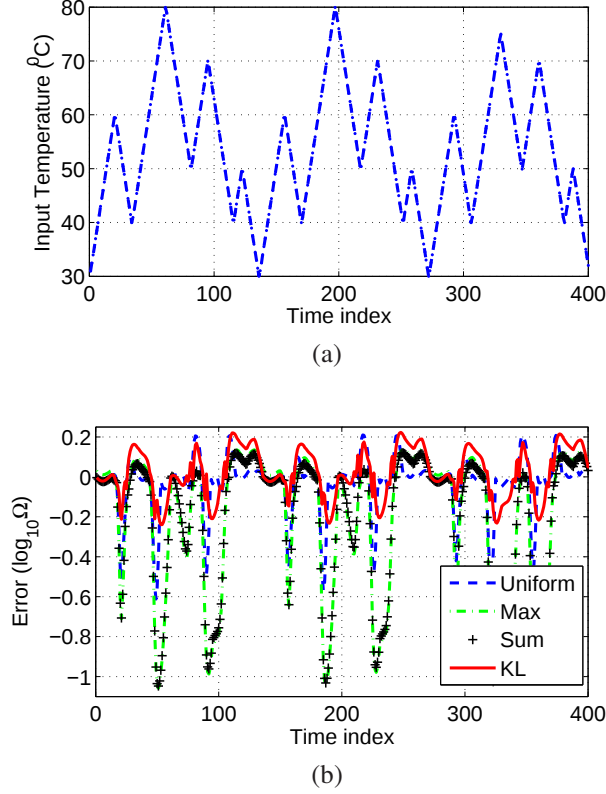
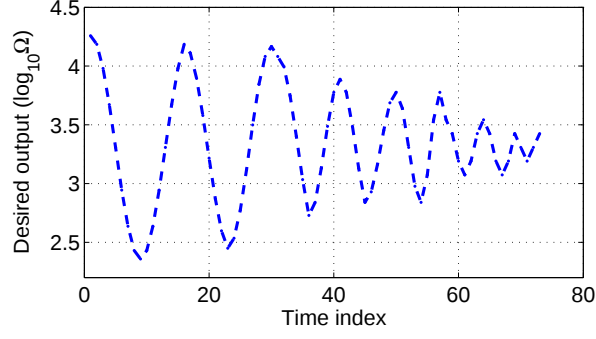


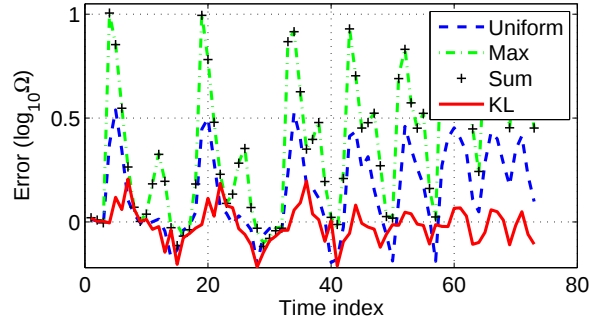
Figure 2.5: (a) Input temperature sequence for model validation. (b) Output error comparison between uniform discretization, maximum of information loss, sum of information loss and KL divergence of information loss.

density function of Preisach operator is evident.

Inverse control performance based on the four models was also examined. Given the desired output sequence, a temperature input that achieves the given output can be calculated iteratively. This calculated input sequence was applied to the system to verify the performance of the identified model. Fig. 2.6(a) shows the desired sinusoidal output with decreasing amplitudes, and Fig. 2.6(b) shows the corresponding inversion errors based on the four different models. It can be seen that the inversion based on the proposed model is effective, with the largest error of $0.22 \log_{10} \Omega$, for the total range of $[-4.28, -2.34] \log_{10} \Omega$. As time evolves the output error almost converges to zero, and since the discretization level is only 4, the inversion performance can be considered very good. In comparison, the inversion of the uniform discretization approach yields errors larger



(a)



(b)

Figure 2.6: (a): Desired output of sinusoidal shape sequence. (b): Inverse compensation errors between uniform discretization, max of information loss, sum of information loss and KL divergence of information loss.

than $0.56 \log_{10} \Omega$, and as large as $1.00 \log_{10} \Omega$ for the other two approaches. This again shows the advantage of the proposed KL divergence-based compression scheme.

Chapter 3

Entropy-based Optimal Compression of the Generalized Prandtl-Ishlinskii (GPI) Model

In this chapter, the optimal compression of the GPI model subject to a given number of play operators is presented. An information-theoretic tool, entropy, is adopted to capture the information loss in replacing a group of weighted play operators with a single play operator. Prior to compression, a scaling operation on the original weights is introduced to accommodate the fact that, given the same weight value, a generalized play with a larger radius has less impact on the total output. The optimal compression algorithm is reformulated as an optimal control problem and solved with dynamic programming, the computational complexity of which is shown to be much lower than that of exhaustive search.

Extensive simulation results are presented to examine the performance of the proposed approach in approximating a GPI model consisting of a large number of play operators, where cases with different types of weight distributions are explored. Simulation results show that, in general, the entropy-based approaches deliver far better performance than a typically adopted scheme where the play radii are assigned uniformly. The effectiveness of the proposed approach is further verified in the compression of an experimentally identified GPI model for the hysteretic relationship between temperature and resistance of a VO_2 film. Note that the proposed optimal compression approach also works for the CPI model since it is a special case of the GPI model.

3.1 Problem Formulation

Note that, for a GPI model, its output is a weighted superposition of outputs (states) of individual play operators and hence is linear with respect to the weight parameters. As a result, one can identify the weight parameters offline or online by minimizing the difference between the actual output and the output of the estimated model. On the other hand, the play radii of a GPI model determines the states of individual play operators, but the relationship between the radii and the states are complex and involves the past history of the input, and one cannot express the output of a GPI model directly in terms of its play radii. Consequently, determining the play radii based on the output difference between the original model and the estimated model is difficult if not impossible. Therefore, in this work we seek to minimize the difference in “weight distribution” (including both play radii and their weights) between the original and reduced GPI operators, which would imply that the output of the reduced model will be close to that of the original model, under all input functions.

The number of play operators in the GPI model directly determines the computational and storage cost in hysteresis modeling, parameter identification, and inverse compensation. Therefore, it is taken as the measure of complexity for a GPI model. Consequently, the compression of a high-fidelity GPI model with a large number (N) of play operators deals with finding a smaller number (M , $M < N$) of play operators and the corresponding weights to best represent the original GPI model. This problem is closely tied to optimal compression of the weight vector $\{p(r_j)\}_{j=1}^N$ in the discrete case, which is formulated precisely.

Fig. 3.1 shows a nonnegative weighting function $p(r)$ with N elements, $p_j = p(r_j)$, $0 \leq r_1 < r_2 < \dots < r_N < \infty$. The compression of the original weighting function is to use a new weighting function with (M , $M < N$) elements: $\hat{p}_j = \hat{p}(\hat{r}_j)$, $0 \leq \hat{r}_1 \leq \hat{r}_2 \leq \dots \leq \hat{r}_{M-1} \leq \hat{r}_M < \infty$ to approximate

the original weighting function.

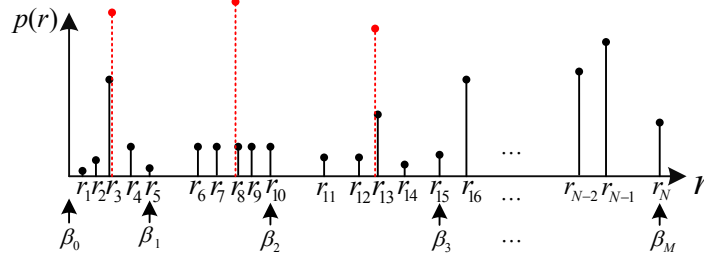


Figure 3.1: Schematic illustrating the compression of a weighting function. The solid-line segments are the original weighting function, and the dotted-line segments are the new weighting function.

In order to compress the weighting function, the notion of partition is firstly introduced. Denote $\mathcal{D} = \{\beta_k\}_{k=0}^M$ as the set of partition indices ($0 = \beta_0 < \beta_1 < \dots < \beta_{M-1} < \beta_M = N$), that partitions the weighting function into M groups. The original weights with indices $\beta_{k-1} + 1, \beta_{k-1} + 2, \dots, \beta_k$ belong in the k -th group, $k = 1, 2, \dots, M$. For each group, the original weighting function is approximated with only one element (shown in Fig. 3.1 as red dotted segments), and the new element is characterized as

$$\hat{r}_k = \frac{\sum_{i=\beta_{k-1}+1}^{\beta_k} p(r_i)}{\sum_{j=\beta_{k-1}+1}^{\beta_k} p(r_j)} r_i, \quad (3.1)$$

$$\hat{p}(\hat{r}_k) = \sum_{i=\beta_{k-1}+1}^{\beta_k} p(r_i).$$

The optimal compression problem is to find the compression strategy $\mathcal{D}^* = \{\beta_k^*\}_{k=0}^M$ that best approximates the given weighting function. To facilitate the formulation of the problem, we consider a function F_k as the information loss measure in approximating the distribution $p(r_i), \beta_{k-1} + 1 \leq i \leq \beta_k$ with $\hat{p}(\hat{r}_k)$. The overall compression cost function can be chosen as either $\sum_{k=1}^M F_k$ or $\max_k F_k$.

3.2 Optimal Compression Scheme

While a number of methods, such as evolutionary algorithms [61] and simulated annealing [62], could be used to solve nonlinear optimization problems, these approaches are typically computation-intensive and cannot guarantee globally optimal solutions. In this work, we exploit the structure of the compression problem and reformulate it as an optimal control problem. The reformulation allows us to use dynamic programming to obtain the (globally) optimal solution, as well as analyze the complexity of the algorithm. Denote $x_k = \beta_k$ as the state, and $u_k = \beta_k - \beta_{k-1}$ as the control input, $k = 1, \dots, M$. The optimization problem is then reformulated as: finding inputs $\mathbf{u} = (u_1, u_2, \dots, u_{M-1})$, such that the total cost is minimized. Note that since $\beta_0 = 0$ and $\beta_M = N$ are fixed, u_M will be determined automatically by $\mathbf{u}$ and thus is not a decision variable. The compression cost F_k for each group is clearly determined by β_{k-1} and u_k , or $F_k = F_k(x_{k-1}, u_k)$. The dynamic programming algorithm to be presented next considers the overall cost function J_1 with the form of $\sum_{i=1}^M F_i$. The algorithm is similar for the case when the cost function is in the form of $\max_i F_i$. Specifically, we have

$$x_k = x_{k-1} + u_k, \tag{3.2}$$

$$J_1(x_0, \mathbf{u}) = \sum_{i=1}^{M-1} F_i(x_{i-1}, u_i) + f(x_{M-1}),$$

where $f(x_{M-1})$ represents the “terminal cost” – the information loss for the last group. The optimal control $\mathbf{u}^* = (u_1^*, u_2^*, \dots, u_{M-1}^*)$ is defined as

$$\mathbf{u}^* = \arg \min_{\mathbf{u}} J_1(x_0, \mathbf{u}). \tag{3.3}$$

Once the optimal control $\mathbf{u}^*$ is found, the optimal compression strategy is obtained as: $\beta_0^* = 0, \beta_1^* = \beta_0^* + u_1^*, \dots, \beta_k^* = \beta_{k-1}^* + u_k^*, \dots, \beta_M^* = N$. The following proposition provides the algorithm for finding $\mathbf{u}^*$, the proof of which follows the standard dynamic programming principle [63].

Proposition 1. *Consider a sequence of $M - 1$ optimization problems, with the cost functions defined as*

$$J_k(x_{k-1}, \{u_i\}_{i=k}^{M-1}) = \sum_{i=k}^{M-1} F_i(x_{i-1}, u_i) + f(x_{M-1}), \quad (3.4)$$

$k = 1, \dots, M - 1$, and the corresponding value function as

$$V_k(x_{k-1}) = \min_{\{u_i\}_{i=k}^{M-1}} J_k(x_{k-1}, \{u_i\}_{i=k}^{M-1}). \quad (3.5)$$

Then the value functions along with the optimal control sequence $\{u_i^\}$ can be obtained recursively as follows:*

$$V_{M-1}(x_{M-2}) = \min_{u_{M-1}} F_{M-1}(x_{M-2}, u_{M-1}) + f(x_{M-2} + u_{M-1}), \quad (3.6)$$

$\dots$

$$V_k(x_{k-1}) = \min_{u_k} F_k(x_{k-1}, u_k) + V_{k+1}(x_{k-1} + u_k), \quad (3.7)$$

$k = M - 2, \dots, 2, 1$, and u_k^ is obtained as the minimizing u_k in the computation of $V_k(x_{k-1})$, $k = 1, \dots, M - 1$.*

Remark 1. *Note that the procedure in Proposition 1 will generate $\{u_k^*\}$ as a state-dependent policy. The original optimization problem has a fixed initial state of $x_0 = 0$, which results in a specific optimal control sequence when applied to the feedback policy.*

The dynamic programming approach has a significant advantage over the exhaustive search in terms of computational complexity. Take the number of evaluations of information loss in partitioned groups required by each algorithm as the metric of computational complexity. For the dynamic programming approach, the terminal cost function $f(x_{M-1})$ requires N evaluations since x_{M-1} could take any values of $\{0, 1, \dots, N-1\}$, (3.6) requires $N-1$ evaluations, and (3.7) requires $N-M+k$ evaluations, $1 \leq k \leq M-2$, resulting in a total of $\frac{M(2N-M+1)}{2}$ evaluations. For the exhaustive search, on the other hand, there are $\frac{(N-1)!}{(M-1)!(N-M)!}$ possible partitions for the weights, and each partition requires M evaluations, resulting in a total of $\frac{(N-1)!M}{(M-1)!(N-M)!}$ for the number of evaluations, which is significantly larger than the complexity of the dynamic program algorithm.

3.3 Information Loss Metrics: Entropy-based Measure

The discussions so far have assumed a generic function F_k that represents the information loss in replacing the weight distribution of the k th group, $\{p(r_i)\}_{i=\beta_{k-1}+1}^{\beta_k}$, with a single weight $\hat{p}_k(\hat{r}_k)$. An information-theoretic tool, entropy, is exploited to define the information loss in compression.

Entropy [64] is a measure of the uncertainty in a random variable, which has been used extensively in statistics [65] and signal processing [66]. For a discrete random variable G with probability mass function (pmf) $\bar{p}(r_i)$, $i = 1, 2, \dots, L$, the entropy is defined as

$$H(G) = - \sum_{i=1}^L \bar{p}(r_i) \log(\bar{p}(r_i)). \quad (3.8)$$

The convention $0 \log 0 = 0$ is adopted. For a given L , the entropy of G is lowest when there exists a $k \leq L$, such that $\bar{p}(r_k) = 1$. On the other hand, the uniform distribution, where $\bar{p}(r_i) = 1/L$, $i = 1, 2, \dots, L$, has the largest entropy.

Intuitively, if the weight distribution of (multiple) play operators, when properly normalized, is close to a uniform distribution, the compression loss is high; Conversely, if the group has a single operator with weight dominantly larger than those of the rest operators, the compression loss is expected to be small. These considerations make the entropy a natural candidate for measuring the information loss. In addition, if the dominant play operators are located far away from $\hat{r}_k$, the compression loss is also high, motivating the incorporation of the distances between the play radii and the “centroid” $\hat{r}_k$ into the cost function. Specifically, the following procedure is proposed to compute an entropy-based measure for the information loss in approximating a discrete distribution group $p(r_i)$, $i = \beta_{k-1} + 1, \beta_{k-1} + 2, \dots, \beta_k$:

1. Calculate the total weight in the group:

$$T_k = \sum_{i=\beta_{k-1}+1}^{\beta_k} p(r_i).$$

2. Get the normalized pmf for the group:

$$\begin{aligned} \bar{p}_i &= p(r_i)/T_k, \\ i &= \beta_{k-1} + 1, \beta_{k-1} + 2, \dots, \beta_k. \end{aligned}$$

3. Obtain the entropy for the normalized pmf:

$$H_k = - \sum_{i=\beta_{k-1}+1}^{\beta_k} \bar{p}_i \log \bar{p}_i.$$

4. The effect of the distances between play radii and the centroid needs to be incorporated; one way to do this is to define the cost function for the k -th group as

$$\mathcal{E}_k = T_k \cdot \sqrt{\sum_{i=\beta_{k-1}+1}^{\beta_k} (\bar{p}_i (r_i - \hat{r}_k))^2 \cdot H_k},$$

where T_k is included to reflect the impact of the total weight for the group.

Note that while there might be other alternatives, we will show later in this work that the proposed scheme is adequately effective. Finally, for a partition strategy $\mathcal{D}$, the entropy-based overall cost functions can be chosen as:

$$J_{\mathcal{E}_{SUM}}(\mathcal{D}) = \sum_{k=1}^M \mathcal{E}_k, \quad (3.9)$$

$$J_{\mathcal{E}_{MAX}}(\mathcal{D}) = \max_k \mathcal{E}_k. \quad (3.10)$$

The optimization algorithms based on the cost functions (3.9) and (3.10) are denoted as Entropy Sum and Entropy Max, respectively.

3.4 Scaling of the Weights for the GPI Model

For a GPI model with a certain input range, the constituent play operators will have different ranges of outputs, and thus have different levels of importance to the output of the GPI model even if their weights are equal. Proper “scaling” of the weighting function is introduced to accommodate the play radius-dependent importance.

Fig. 3.2 shows a generalized play operator with radius r , where the input range is $[v_{\min}, v_{\max}]$, and the initial condition $w(0) = \gamma_L(v_{\min}) + r$. It can be easily seen that the output range of the play operator is dependent on r ; specifically, the output $w \in [\gamma_L(v_{\min}) + r, \gamma_R(v_{\max}) - r]$, with a total change of $\gamma_R(v_{\max}) - \gamma_L(v_{\min}) - 2r$. Accordingly, the following scheme is introduced to produce a “scaled” weight distribution for the compression.

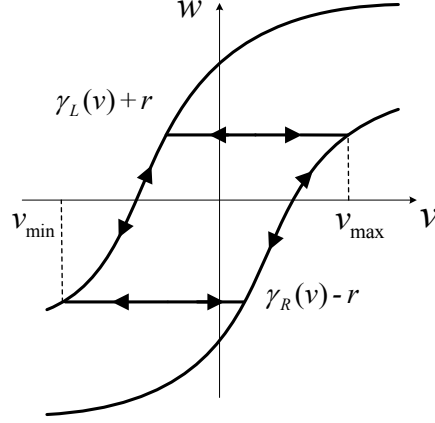


Figure 3.2: Illustration of the radius-dependent output range for a generalized play operator.

Denote the actual weight as p , and the weight after scaling as p' , then for the play operator with radius r_j :

$$p'(r_j) = \frac{\gamma_R(v_{\max}) - \gamma_L(v_{\min}) - 2r_j}{2} * p(r_j). \quad (3.11)$$

For a generalized play operator whose envelopes are in the form of hyperbolic-tangent functions, when $v_{\min} \rightarrow -\infty$ and $u_{\max} \rightarrow +\infty$, the output $z \in [-1 + r, 1 - r]$. It can be seen that $0 \leq r \leq 1$. The play operator will not produce any output change under a cyclic input when the radius $r > 1$, since the output will never reach both envelopes due to the disjoint ranges of the envelopes. It is for this reason that the radius is always chosen to be no larger than 1. The advantage of using the scaled weights over the non-scaled weights in compression will be further demonstrated.

3.5 Simulation Results

The proposed optimal compression algorithms are tested in simulation for GPI models with different characteristics for their scaled weighting functions.

Following [13, 28], the envelope functions for the generalized play operator are chosen to be

Table 3.1: Parameters of the GPI model envelope functions.

a_R	b_R	a_L	b_L
3.5	0	4.5	1

hyperbolic-tangent functions in the form of

$$\gamma_R(v(t)) = \tanh(a_R v(t) + b_R), \quad (3.12)$$

$$\gamma_L(v(t)) = \tanh(a_L v(t) + b_L). \quad (3.13)$$

For simplicity of demonstration, the non-hysteretic component $D(v(t))$ is set to be zero. The parameter values of the generalized play operator are shown in Table 3.1. The original GPI model consists of $N = 30$ play operators, with radii $r_j = j/(N + 1)$, $j = 1, 2, \dots, N$, and input range of $v \in [-1, 1]$.

The compression goal is to use a new GPI model with $M = 6$ play operators to approximate the original GPI model. Although the unscaled weights are directly related to the output, the scaled weight distribution is considered. The output performance of the proposed approach will be discussed. Four cases for the scaled weight distribution of the original model are considered, 1) uniform, 2) one peak, 3) two peaks, and 4) random. In addition to the two compression schemes presented in the previous section (Entropy Sum, Entropy Max), a uniform compression scheme, where every five consecutive play operators are clustered into one group, is considered for comparison purposes.

In order to assess the output prediction performance of the reduced model, throughout the chapter, the normalized RMSE is adopted to quantify the modeling performance under different compression strategies. The error is obtained as follows: first, calculate the RMSE between the output

Table 3.2: Compression performance comparison: the uniform case.

Scheme	Partition indices	Error
Uniform	(0,5,10,15,20,25,30)	0.75%
Entropy Sum	(0,5,10,15,20,25,30)	0.75%
Entropy Max	(0,5,10,15,20,25,30)	0.75%

of each new GPI model and that of the original GPI model under the input shown in Fig. 3.3(a); then divide the RMSE by the output range of the original model. Normalization of the RMSE allows assessing and comparing the algorithms' performance across different weight distributions.

3.5.1 Case 1: Uniform Distribution for the Scaled Weights

First, the following uniform distribution is considered for the scaled weights:

$$p'(r_j) = 0.5, j = 1, 2, \dots, 30. \quad (3.14)$$

Fig. 3.3(a) shows the actual weight distribution (unscaled) and Fig. 3.3(b) shows the corresponding scaled weight distribution. Fig. 3.4(a) shows an input sequence and Fig. 3.4(b) shows the input-output relationship of the given GPI model. Note that the actual weighting function and the input-output relationship will not be shown for other forms of weighting functions in the interest of brevity; however, the hysteresis loops in other cases are also verified to be large.

The simulation results are summarized in Table 3.2. It is shown that, given the uniform distribution, the two entropy-based algorithms are able to compress the distribution uniformly, and generate desirable performance.

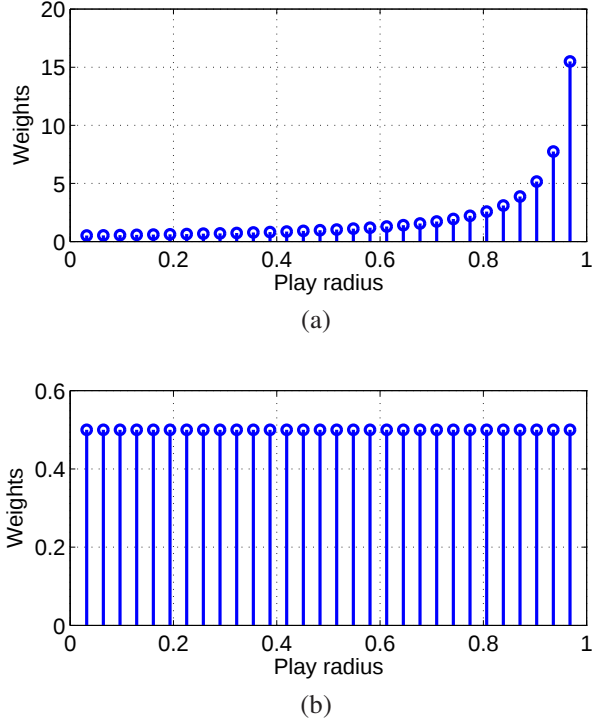


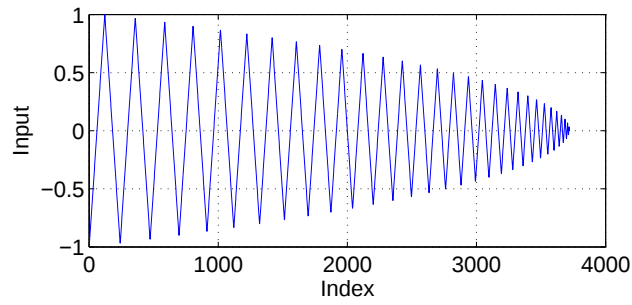
Figure 3.3: Weighting function (uniform case) of the GPI model: (a) Unscaled. (b) Scaled.

3.5.2 Case 2: Scaled Weights with One Prominent Peak

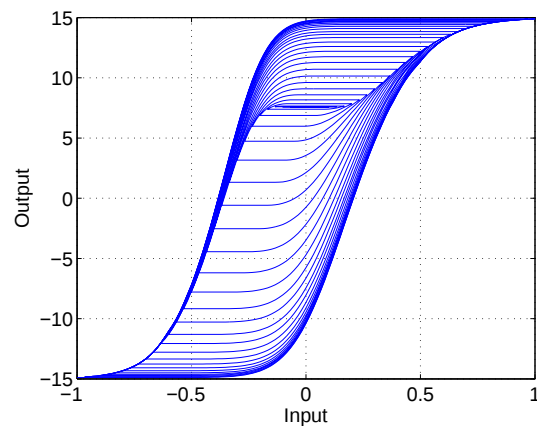
In the second case, the scaled weight distribution is assumed to have one peak, expressed as:

$$p'(r_j) = \frac{5}{\sqrt{2\pi}} \exp\left(-\frac{(j-15)^2}{15}\right), j = 1, 2, \dots, 30. \quad (3.15)$$

Fig. 3.5 shows the scaled weight distribution. Table 3.3 shows the compression performances based on different approaches. From the simulation results, it is seen that Entropy Sum and Entropy Max approaches are able to generate considerably better performance than the uniform compression scheme. The peak of the original weighting function is in the middle region; the simulation results show that the entropy approaches partition the weights densely in the middle region (with many groups having only one or two elements).



(a)



(b)

Figure 3.4: (a) Input sequence. (b) Input vs output for the GPI model with uniform weight function.

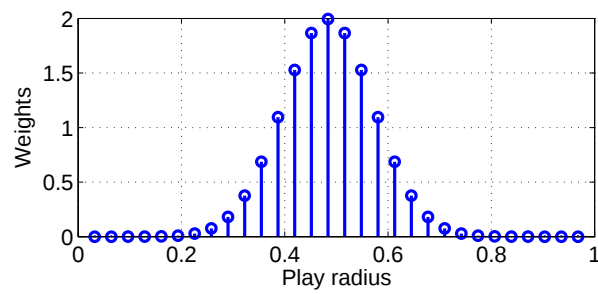


Figure 3.5: Scaled weighting function (one-peak case).

Table 3.3: Compression performance comparison: the case of one peak.

Scheme	Cut indices	Error
Uniform	(0,5,10,15,20,25,30)	0.34%
Entropy Sum	(0,11,13,14,16,18,30)	0.08%
Entropy Max	(0,9,12,14,16,18,30)	0.14%

3.5.3 Case 3: Scaled Weights with Two Prominent Peaks

In the third case, the scaled weighting function has two peaks, expressed as:

$$p'(r_j) = \begin{cases} \frac{5}{\sqrt{2\pi}} \exp(-\frac{(j-8)^2}{8}), & j = 1, 2, \dots, 16 \\ \frac{5}{\sqrt{2\pi}} \exp(-\frac{(j-23)^2}{8}), & j = 17, 18, \dots, 30. \end{cases} \quad (3.16)$$

Fig. 3.6 shows the scaled weight distribution. Table 3.4 shows the compression performances based on the different compression approaches. From the simulation results, both of the proposed approaches show very good performance, with about 40% less error comparing to the uniform compression scheme.

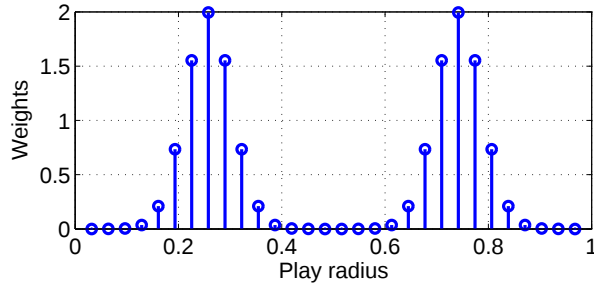


Figure 3.6: Scaled weighting function (two-peak case).

3.5.4 Case 4: Random Distribution for the Scaled Weights

Finally, we consider the case where the scaled weighting function has a random distribution as shown in Fig. 3.7. Table 3.5 lists the corresponding simulation results. It can be seen that, under

Table 3.4: Compression performance comparison: the case of two peaks.

Scheme	Partition indices	Error
Uniform	(0,5,10,15,20,25,30)	0.42%
Entropy Sum	(0,7,8,14,22,23,30)	0.27%
Entropy Max	(0,6,8,11,21,23,30)	0.20%

Table 3.5: Compression performance comparison: the case of random distribution.

Scheme	Partition indices	Error
Uniform	(0,5,10,15,20,25,30)	0.53%
Entropy Sum	(0,5,9,13,18,24,30)	0.54%
Entropy Max	(0,6,10,15,20,25,30)	0.53%

a random distribution, the entropy approaches compress the distribution almost uniformly, with slightly better performance than the uniform compression scheme. A random distribution is similarly difficult as a uniform distribution to compress, since there are usually no particular patterns that facilitate compression.

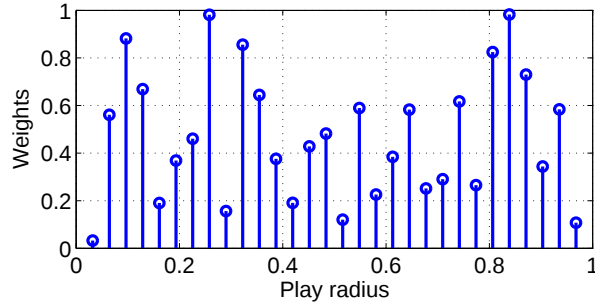


Figure 3.7: Scaled weighting function (the random case).

From the simulation results, overall, both of the proposed approaches show good compression performance. When the pattern of the (scaled) weighting function is uniform or random, the optimal compression almost degenerates to uniform compression, and the compression error is larger comparing with other cases that have more features (peaks).

3.5.5 Computational Time for the Algorithms

The computational time of the dynamic programming-based optimization is also compared with that using exhaustive search. Due to the similar optimization process under different cost function candidates, only Entropy Sum is considered in this comparison. A GPI model with $N = 30$ is used, which has a scaled weight distribution as used in Section 3.5.2. The computations are run in Matlab on a computer Lenovo Thinkpad T420 with 2.80 GHz CPU and 4.00 GB memory.

In order to compare the computational efficiency, the dynamic programming-based algorithm and the exhaustive search-based algorithm are run 10 times for each setting of M , which is varied from 2 to 7 in this study. The average running times are shown in Fig. 3.8. It can be seen that, when N is fixed, the time cost under dynamic programming grows much slower than the exhaustive search when M is increased. These results agree well with the complexity analysis in Section 3.2, as shown in Fig. 3.9, which plots the number of information loss evaluations for the dynamic programming and the exhaustive search methods, respectively. The computational advantage of the dynamic programming approach is evident.

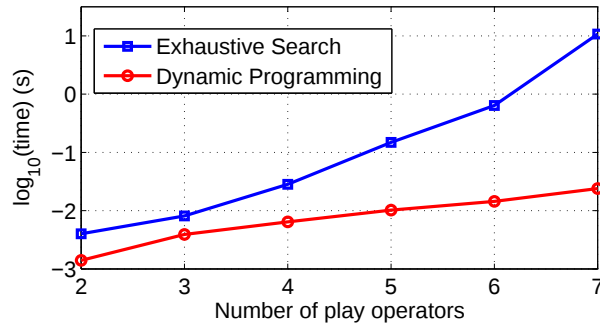


Figure 3.8: Comparison of average optimization time. Note the log scale.

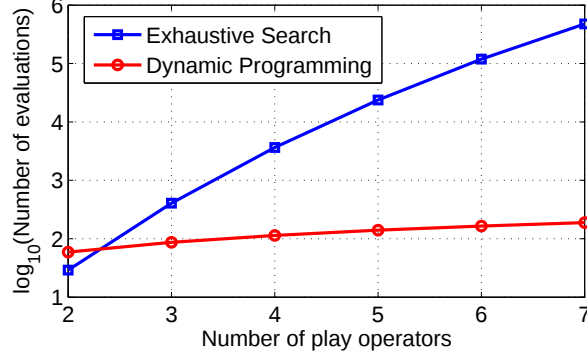


Figure 3.9: Comparison of the number of information loss evaluations. Note the log scale.

3.5.6 Comparison with a Traditional Model Identification Approach

The effectiveness of the proposed optimal compression approach is further compared with a traditional model identification scheme (referred to as “output optimization” in this work), where a model with the same complexity (six generalized play operators) is determined by minimizing the output error under a given training input. While there are infinite number of possible choices for the training input, a third-order reversal input sequence (shown in Fig. 3.10(a)) is adopted as a representative example. An extensive search within all possible parameterizations of the 6 play operators are conducted in Matlab using the function *fmincon*, to match the output of the original model with 30 plays. The scaled weighting function for the original model has the same random case as shown in Fig. 3.7 and the corresponding output sequence is shown in Fig. 3.10(b). Fig. 3.10(c) shows the corresponding output prediction error under the Entropy Sum approach and the output optimization approach. The RMSE errors of the Entropy Sum approach and the output optimization approach are 0.165 and 0.088, respectively. While that latter indicates the output optimization approach could deliver better performance for a given input sequence, Fig. 3.11 shows that the proposed approach is more robust in output prediction with respect to input variability. In particular, simulation is run 50 times with different, randomly generated input sequences and the corresponding output prediction performance is recorded. Fig. 3.11(a) and (b) shows one example

Table 3.6: Identified parameters of the envelope functions.

a_R	b_R	a_L	b_L	a_D	b_D	d
0.201	-11.578	0.162	-10.262	0.029	-1.611	-3.257

for the 50 cases, while Fig. 3.11(c) summarizes the error statistics over the 50 runs.

3.6 Experimental Results

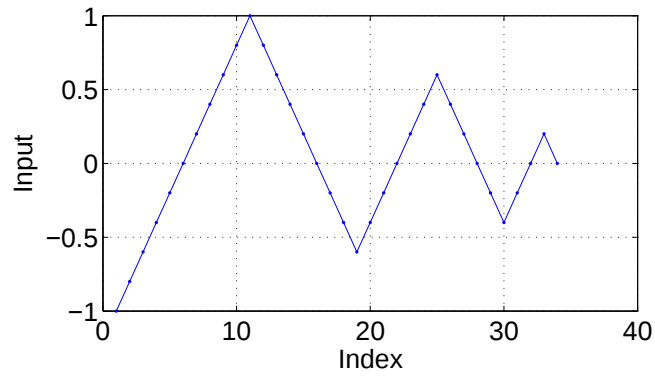
The hysteresis between resistance and temperature of a VO₂ film was used as an example to show the effectiveness of the proposed discretization scheme.

In order to get desirable modeling performance, the original GPI model has $N = 30$ play operators. Similarly, their play radii are $r_j = j/N$, $j = 1, 2, \dots, N$, respectively, and the envelope functions for the generalized play operator are chosen to be hyperbolic-tangent functions in the form of Eq. (3.12) and Eq. (3.13).

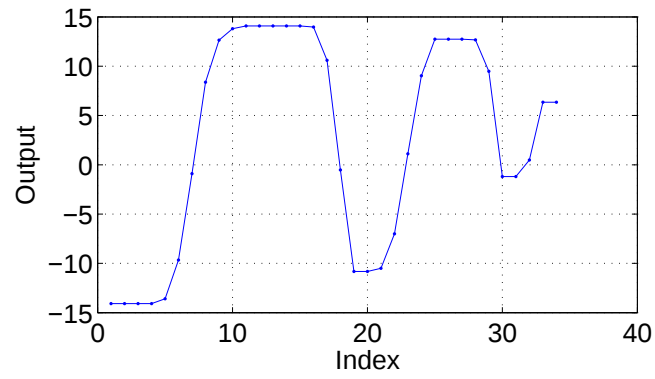
The non-hysteretic component $D(v(t))$ is chosen to be

$$D(v(t)) = \tanh(a_D v(t) + b_D) + d. \quad (3.17)$$

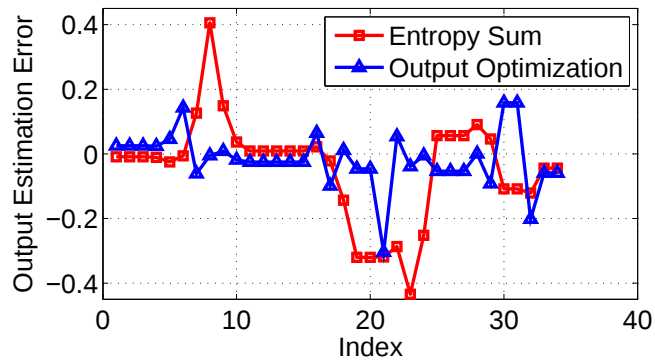
The full range of temperature input is $[30, 90]^\circ\text{C}$. The hysteresis behavior shown in Fig. 3.12 is asymmetric and partially saturated. The GPI model is identified based on the approach in [13]. The effectiveness of the GPI model is verified in Fig. 3.12. Table 3.6 and Fig. 3.13(a) show the identified parameters for the envelope functions and the weights of the generalized play operators, respectively. Fig. 3.13(b) shows the weight after scaling based on the actual weight. The weights present a non-uniform distribution.



(a)

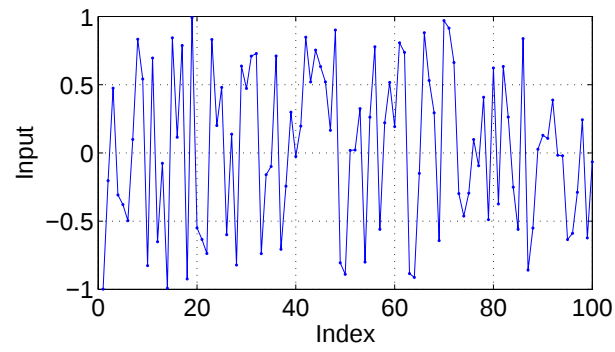


(b)

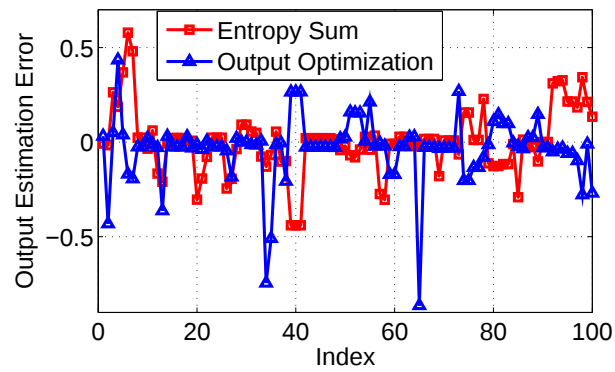


(c)

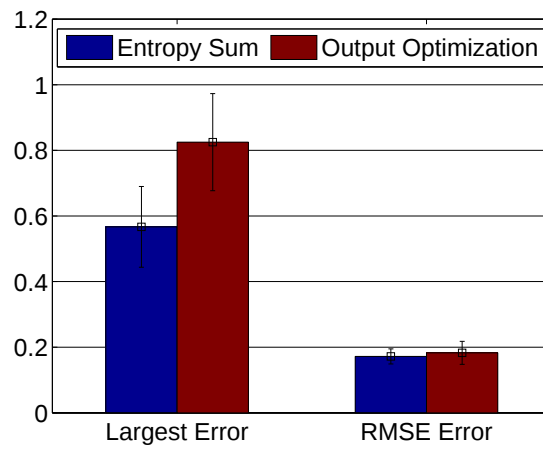
Figure 3.10: (a) A third-order reversal input sequence. (b) The corresponding output sequence. (c) The output prediction error between the Entropy Sum approach and output optimization approach.



(a)



(b)



(c)

Figure 3.11: (a) A random input sequence, and (b) the corresponding output prediction error performance. (c) The output prediction performance based on 50 random input sequences.

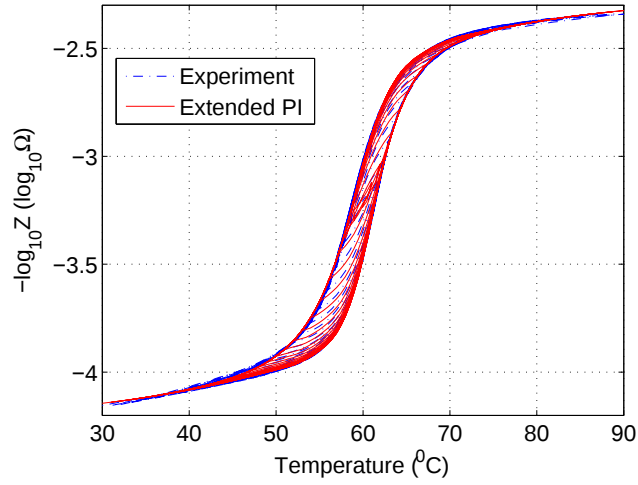


Figure 3.12: The performance of a GPI model (30 plays) in modeling of the resistance-temperature hysteresis in VO_2 .

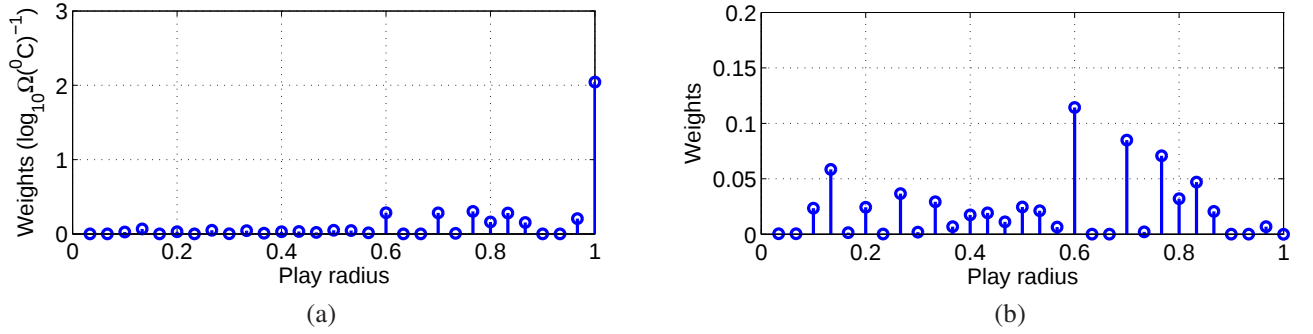


Figure 3.13: (a) Identified weights for all the play operators of the GPI model. (b) The scaled weights for the GPI operators.

3.6.1 Compression Performance

To conduct the compression studies, the identified GPI model is taken as the “original” distribution. The new GPI model consists of $M = 5$ play operators. The partition scheme under uniform compression was $\{0,6,12,18,24,30\}$. Fig. 3.14 shows the play radii and weights for the M play operators. Uniform compression fails to accommodate the weighting distribution, with an RMSE of 1.10% for the modeling error.

The partition scheme under Entropy Sum was $\{0,7,14,20, 23,30\}$, and that under Entropy Max

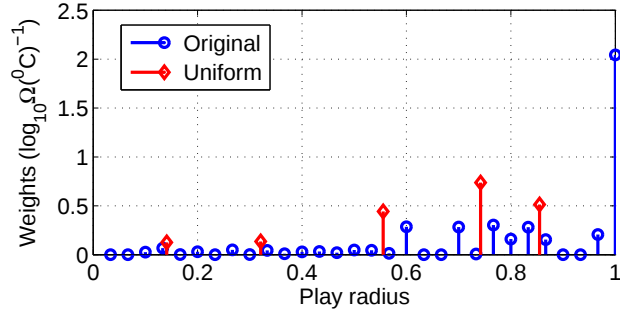


Figure 3.14: Parameters of the compressed GPI model: uniform compression.

Table 3.7: Modeling verification error comparison.

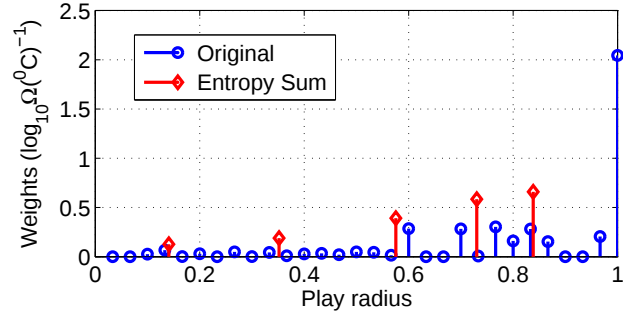
Scheme	Non-scaled distribution	Scaled distribution
Uniform	1.76%	1.45%
Entropy Sum	1.21%	1.05%
Entropy Max	0.87%	0.72%

was $\{0,4,10,17,22, 30\}$. Fig. 3.15(a),(b) show the compressed play operator radii and weights based on Entropy Sum and Entropy Max, respectively. Both schemes worked much better than the uniform compression case, with RMSE values of 0.73% and 0.76%, respectively, which were about 32% smaller than that in the uniform case.

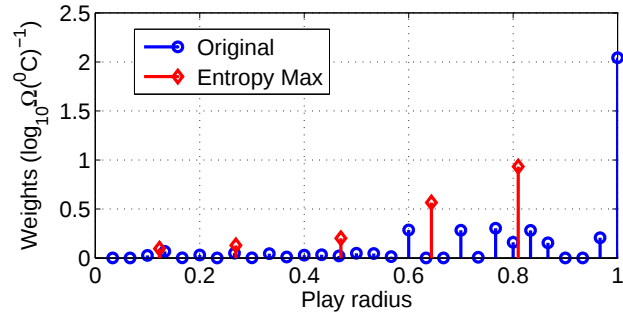
3.6.2 Model Verification

In order to further validate the proposed approach, a randomly chosen temperature input sequence, shown in Fig. 3.16(a), was applied to the VO_2 film, and the corresponding resistance output was measured as shown in Fig. 3.16(b). Predictions of the resistance output were obtained based on the compressed GPI models obtained with different schemes. The corresponding estimation errors were calculated and shown in Fig. 3.16(c) and Table 3.7.

The model verification experiments further demonstrate that the proposed compression schemes outperform the uniform compression. In Table 3.7, the modeling performance without considering



(a)



(b)

Figure 3.15: Parameters of the compressed GPI model: (a). Entropy Sum. (b) Entropy Max.

the scaling effect is also included [67]. It is evident that the performance improves with proposed scaling strategy; the results improve about 10-20% with the scaling.

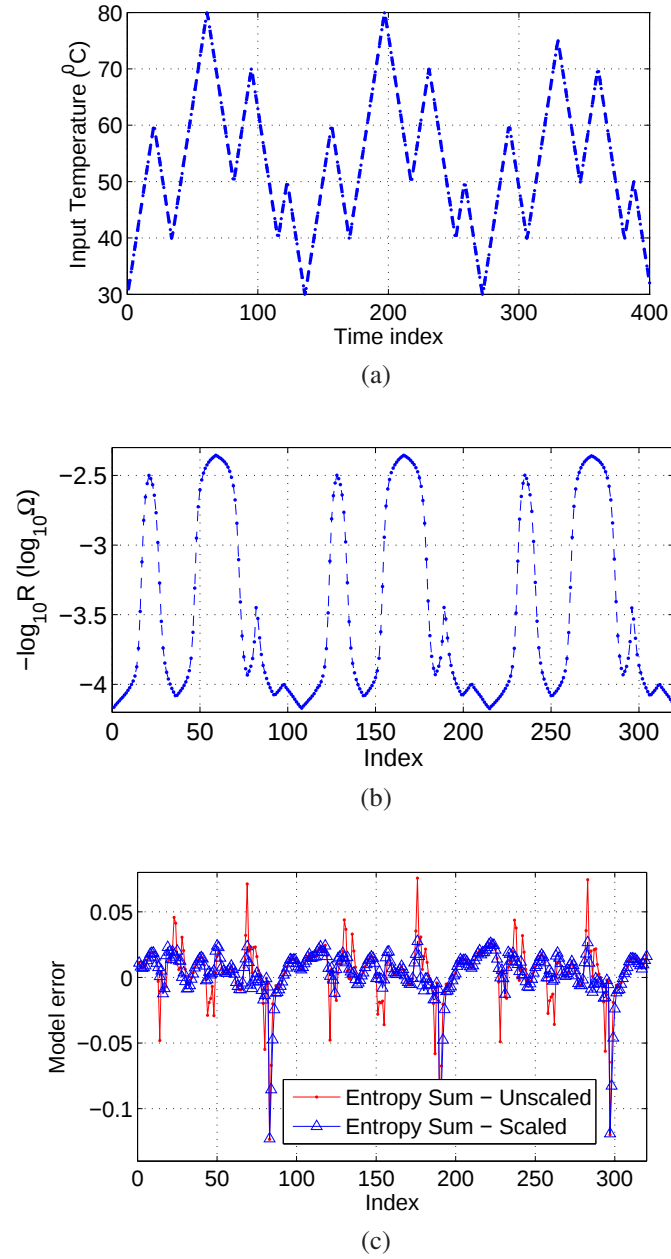


Figure 3.16: (a) A new temperature input sequence for model verification. (b) Corresponding output sequence. (c) The output prediction error comparison of Entropy Sum Unscaled approach and the Entropy Sum Scaled approach.

Chapter 4

Compressive Sensing-based Preisach Operator Identification

In this chapter, identification of the Preisach operator is studied under the compressive sensing framework that requires much fewer measurements. The proposed approach adopts the DCT transform of the output data to obtain a sparse vector. The model parameters can be efficiently reconstructed using the proposed scheme. Sparser coefficients are obtained assuming the order of all the output data are known, and a constraint least-squares method is further adopted to ensure the reconstructed density vector contains no negative elements. The least-squares scheme has been also realized, and is compared with the proposed approach. RMSE error is adopted to examine the identified model parameters and model estimation performances. The proposed identification approach has been shown to have better identification performance than the least-squares scheme through both simulation and experiments involving a VO₂-integrated microactuator.

4.1 Problem Formulation

Consider the Preisach operator $\Gamma[v(\cdot); \zeta_0](t)$

$$u(t) = \Gamma[v(\cdot); \zeta_0](t) = \int_{\mathcal{D}_0} \mu(\beta, \alpha) \gamma_{\beta, \alpha}[v(\cdot); \zeta_0(\beta, \alpha)](t) d\beta d\alpha, \quad (4.1)$$

where v is the input, ζ_0 denotes the initial condition, $\mu \geq 0$ is the density function, $\mathcal{P} = \{(\beta, \alpha) : \beta \leq \alpha\}$ is the *Preisach plane*. The density is approximated by a piecewise constant function—the density value μ_{ij} is constant within cell (i, j) , $i = 1, 2, \dots, L$; $j = 1, 2, \dots, L - i + 1$ [12]. An example of Preisach operator density function discretization is shown in Fig. 4.1. Note that the cells on the diagonal are assumed to have the same area as other cells in this chapter.

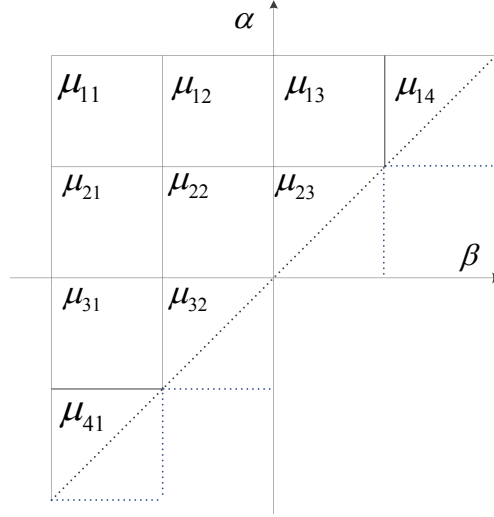


Figure 4.1: Illustration of a discretization of the Preisach density function, where the discretization level $L = 4$.

The output of the Preisach operator (in the discrete-time setting) at time n is written as

$$\tilde{u}(n) = \mu_0 + \sum_{i=1}^L \sum_{j=1}^{L+1-i} \mu_{ij} s_{ij}(n), \quad (4.2)$$

where μ_0 is a bias constant, μ_{ij} is the density value for cell (i, j) , and $s_{ij}(n)$ denotes the *signed* area of the cell (i, j) , representing the accumulative effect brought by all the hysterons within cell (i, j) .

To simplify the discussion, write all the model parameters into a column vector

$$w = \begin{pmatrix} w_1 & w_2 & \cdots & w_{L(L+1)/2} & \mu_0 \end{pmatrix}^\top,$$

where $w_k = \mu_{ij}$, $k = (i-1)(2L-i+2)/2 + j - 1$. Apply an input sequence $v[n]$, $n = 1, 2, \dots, N$, with sufficient excitation and then determine the corresponding $s_{ij}[n]$ by tracking the evolution of the memory curve on the Preisach plane. Stack $s_{ij}[n]$ into a row of a matrix: $S(n, k) = s_{ij}(n)$, and $S(n, L(L+1)/2 + 1) = 1$. The output vector of the model,

$$\tilde{u} = \begin{pmatrix} \tilde{u}(1) & \tilde{u}(2) & \dots & \tilde{u}(N) \end{pmatrix}^\top,$$

can be expressed as

$$\tilde{u} = Sw. \tag{4.3}$$

Assume that the measured output under $v[n]$ is expressed as

$$b = \begin{pmatrix} b(1) & b(2) & \dots & b(N) \end{pmatrix}^\top.$$

The parameters w can be determined such that $\|Sw - b\|_2$ is minimized with the non-negative constraint imposed on all density values [25]. This approach is denoted as “Least Squares”.

When the input sequence for the Preisach operator identification is chosen in the form of damped oscillations, as shown in Fig. 4.2 as an example for $L = 30$, the input sequence is known to produce sufficient excitation for all the density values [12]. The input levels are right at the cell walls, namely, each cell will be either 1 or -1 in terms of the signed area. This particular input sequence is denoted as the “damped oscillation” input sequence. The number of input values equals to the number of model parameters ($N = L(L+1)/2 + 1$). It can be proved that the corresponding S is a full-rank $N \times N$ matrix. Note that only the “damped oscillation” sequence is considered in this chapter, other input sequences also exist such that the corresponding S is a full-rank $N \times N$

matrix.

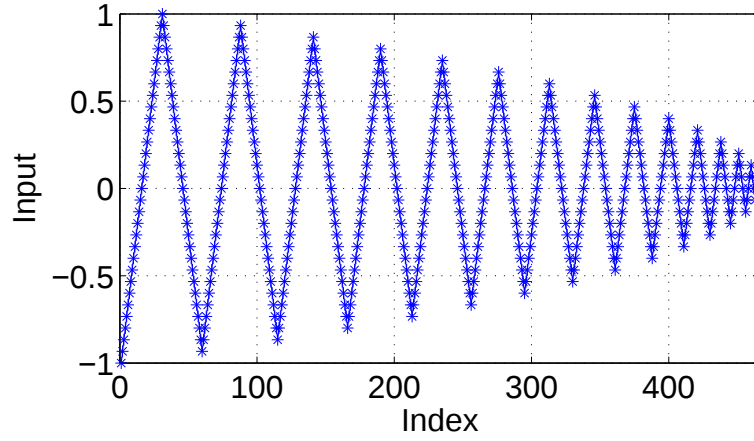


Figure 4.2: The “damped oscillation” sequence for Preisach operator identification ($L = 30$).

When only $M < N$ output measurements y_b are available

$$y_b = \begin{pmatrix} b(n_1) & b(n_2) & \cdots & b(n_M) \end{pmatrix}^\top = Ab,$$

$1 \leq n_1, n_2, \dots, n_M \leq N$, where A is an $M \times N$ matrix whose M rows are randomly chosen from N rows of an $N \times N$ identity matrix, then

$$y_b = A \cdot Sw. \tag{4.4}$$

The number of measurements is less than the number of model parameters. The goal of this work is to faithfully identify the Preisach operator weightings w based on limited measurements y_b .

4.2 Compressive Sensing Scheme for Identifying the Preisach Operator

4.2.1 Overview of Compressive Sensing

Compressive sensing is an alternative to Nyquist-Shannon sampling theory for acquisition and reconstruction of sparse signals. The compressive sensing theory [68–71] states that any length- N signal q that can be well approximated using K coefficients can be faithfully recovered from $M = O(K \log(N/K))$ random linear projections of the signal. Practically, many natural and man-made signals are sparse or compressible in the sense that they have compact representations in a transformed domain, through discrete Fourier transform (DFT) [72], DCT [73], and discrete Wavelet transform (DWT) [74], etc. For example, in [73], audio signals were transformed using one-dimensional DCT, and the sparse DCT coefficients were reconstructed using a compressive sensing-based algorithm. The compressive sensing technique has been successfully applied in signal processing [74–76], networks [77, 78], machine learning [79], as well as system and control [72, 80]. However, there has been little work, if any at all, reported on the use of compressive sensing in hysteresis model identification.

The compressive sensing theory [68–71] states that, if a length- N signal p is K -sparse, which means it contains no more than K non-zero entries, then it is possible to faithfully recover p from its $M = O(K \log(N/K)) \ll N$ random linear projections. In other words, consider

$$y = A\Phi p, \tag{4.5}$$

where y is an $M \times 1$ vector of observations, A is an $M \times N$ measurement matrix, Φ is an $N \times N$ basis transform matrix, and p is an $N \times 1$ K -sparse signal to be recovered. It is proven that the sparse

signal p can be recovered if the matrix $A\Phi$ satisfies the following restricted isometry property (RIP) condition [70],

$$(1 - \delta_S)\|p\|_2^2 \leq \|A\Phi p\|_2^2 \leq (1 + \delta_S)\|p\|_2^2, \quad (4.6)$$

for all S -sparse signal p , where δ_S is the smallest isometry constant of matrix $A\Phi$. Based on [68], p can be recovered efficiently by solving the following l_1 minimization problem,

$$\arg \min \|p\|_1 \quad \text{subject to} \quad y = A\Phi p. \quad (4.7)$$

When $A\Phi$ is a randomly sampled Gaussian matrix, Bernoulli matrix, or Fourier matrix, it has shown to satisfy the RIP condition with very high probability [69]. Practically, many natural and man-made signals are sparse or compressible in the sense that they have compact representations under DFT [72] or DCT [73].

The most common DCT definition [73] for 1-dimensional signal $x_1, x_2, \dots, x_N$ is

$$X_d = \sum_{l=1}^N x_l \cos \frac{\pi}{N} (l + \frac{1}{2})d, \quad (4.8)$$

where $d = 1, 2, \dots, N$. The resulting $N \times N$ DCT matrix Ψ is orthogonal, whose elements can be written as

$$\Psi_{dl} = \cos \frac{\pi}{N} (l + \frac{1}{2})d. \quad (4.9)$$

4.2.2 Compressive Sensing for the Preisach Operator

A novel compressive sensing-based approach for identifying the Preisach operator based on partial output measurements y_b is proposed. In compressive sensing, a random measurement matrix is usually adopted to faithfully recover the sparse signals. Unfortunately, compressive sensing cannot be applied in the Preisach operator identification directly. First, the matrix S in Preisach operator identification must follow certain patterns due to the Preisach plane structure. Second, since the input sequence for identification must provide sufficient excitation, the flexibility of designing the matrix S is further limited. Finally, the density vector w is not necessarily sparse in its original domain.

When the input sequence for the Preisach operator identification is chosen as “damped oscillation” input sequence shown in Fig. 4.2, the corresponding S is a full-rank $N \times N$ matrix. The following proposition is proposed.

Proposition 2. *Consider a Preisach operator (written in the form of Eq. (4.4) with discretization level L , apply the “damped oscillation” input sequence with $N = L(L + 1)/2 + 1$ elements, by tracking the evolution of the memory curve on the Preisach plane, the corresponding S is a full-rank $N \times N$ matrix.*

Proof. Denote S_L as the matrix S under the damped oscillation input sequence for identifying the Preisach operator with discretization level L . The proposition can be proved by mathematical induction as follows,

1. When $L = 1$,

$$\text{rank}(S_1) = \text{rank} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}_{2 \times 2} = 2; \quad (4.10)$$

2. Assume that under “damped oscillation” input sequence with $N = L(L+1)/2 + 1$ elements,
 $\text{rank}(S_L) = N$;
3. Under damped oscillation input sequence with $Z = (L+1)(L+1+1)/2 + 1 = N + L + 1$
elements, by reordering the rows and columns of corresponding S_{L+1} ,

$$\begin{aligned}
\text{rank}(S_{L+1}) &= \text{rank} \left(\begin{bmatrix} s_{1,1} & \cdots & s_{1,N+L+1} \\ \vdots & \ddots & \vdots \\ s_{N+L+1,1} & \cdots & s_{N+L+1,N+L+1} \end{bmatrix} \right)_{Z \times Z} \\
&= \text{rank} \left(\begin{bmatrix} s_{1,1} & \cdots & s_{1,N+L+1} \\ \vdots & \ddots & \vdots \\ s_{L+1,1} & \cdots & s_{L+1,N+L+1} \\ s_{2L+1,1} & \cdots & s_{2L+1,N+L+1} \\ s_{L+2,1} & \cdots & s_{L+2,N+L+1} \\ s_{2L+2,1} & \cdots & s_{2L+2,N+L+1} \\ \vdots & \ddots & \vdots \\ s_{N+L+1,1} & \cdots & s_{N+L+1,N+L+1} \end{bmatrix} \right)_{Z \times Z} \\
&= \text{rank} \left(\begin{bmatrix} 0 & \cdots & 1 & \vdots \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \cdots & 1 & \vdots \\ \cdots & \cdots & \cdots & (S_L)_{N \times N} \end{bmatrix} \right)_{Z \times Z} \\
&= Z.
\end{aligned} \tag{4.11}$$

The third equation is obtained by rearranging certain columns of the previous matrix, more specifically, by rearranging columns with column index $1, L+2, \dots, 1 + \frac{(l-1)(2L-l+4)}{2}, \dots, 1 + \frac{L(L+3)}{2}$, where $l = 1, 2, \dots, L+1$, to the left of the matrix.

So under “damped oscillation” input sequence, S is a full-rank matrix. ■

Exploiting the fact that many natural and man-made signals are sparse in a transformed frequency domain, write

$$q = \Psi \cdot b = \Psi \cdot Sw, \quad (4.12)$$

where Ψ is an $N \times N$ DCT matrix. q is thus an $N \times 1$ column vector contains the DCT coefficients of Sw . It is found that q is approximately sparse in this work when the all the densities follow uniform distributions independently, partially because of the damped oscillation structure of Sw . Since S is a full-rank matrix, the density values w can be expressed as

$$w = S^{-1}\Psi^{-1}q, \quad (4.13)$$

and Eq. (4.4) can be rewritten as

$$y_b = ASS^{-1}\Psi^{-1}q = A\Psi^{-1}q. \quad (4.14)$$

When Ψ is chosen as the DCT matrix, the sparse signal q can be reconstructed via compressive sensing algorithm. The algorithm l_1 -MAGIC is adopted to efficiently reconstruct the sparse signal q using a generic path-following primal-dual method [68]. The density parameter w can be obtained through Eq. (4.14) afterwards. This approach is denoted as “CS”.

The above compressive sensing-based approach only utilizes M input-output data to reconstruct the density values w . If the order of N output data is also known, denote

$$b_{\text{order}} = \begin{pmatrix} b(l_1) & b(l_2) & \cdots & b(l_N) \end{pmatrix}^\top,$$

such that $b(l_1) \leq b(l_2) \leq \cdots \leq b(l_N)$, then

$$q_{\text{order}} = \Psi \cdot b_{\text{order}} = \Psi \cdot S_{\text{order}} w. \quad (4.15)$$

The reconstruction scheme in [68] can still be adopted to identify the density w . q_{order} is found to be more approximately sparse than q , partially because $S_{\text{order}} w$ is monotonically increasing, and has more concentrated frequency components at low frequencies than that of $S w$. It is verified in simulation and experiments that this approach generates better model reconstruction and model estimation performance. This approach is denoted as “CS Order”.

The density w , as obtained in Eq. (4.14), is not necessarily non-negative. In order to facilitate the inverse compensation [9, 26] for dynamic hysteretic systems based on Preisach operator, the density needs to be non-negative. Based on the aforementioned “CS Order” approach, a constraint least-squares approach is further adopted as follows (denoted as “CS Order Non-negative”)

$$\min_w \|\Psi S_{\text{order}} \cdot w - q_{\text{order}}\|_2^2 \quad \text{where } w \geq 0. \quad (4.16)$$

When signal q is K -sparse and the measurements are without any noise, the density may be reconstructed with high accuracy. However, in simulation and experiments, q is found to be approximately sparse. In practical applications, the measurements are with measurement errors and noises. It is thus of practical importance to study the reconstruction performance for approximately

sparse signals with measurement noises.

Proposition 3. *Consider identifying the density w of a Preisach operator using compressive sensing algorithm [68], based on the expression of Eq. (4.14). The measurement noise ε satisfies $\|A\Psi S_{\text{order}} \cdot w - y_b\|_2 \leq \varepsilon$, and let q_s be the truncated signal corresponding to the s largest absolute values of q , then the reconstruction of density w^* obeys*

$$\|w^* - w\| \leq C_{1,s} \cdot \varepsilon + C_{2,s} \cdot \frac{\|q - q_s\|_1}{\sqrt{s}}, \quad (4.17)$$

where $C_{1,s}$, and $C_{2,s}$ are positive constants.

Proof.

$$\begin{aligned} \|w^* - w\|_2 &= \|S_{\text{order}}^{-1} \Psi^{-1} q^* - S_{\text{order}}^{-1} \Psi^{-1} q\|_2 \\ &\leq \|S_{\text{order}}^{-1} \Psi^{-1}\|_2 \cdot \|q^* - q\|_2 \\ &\leq \|S_{\text{order}}^{-1} \Psi^{-1}\|_2 \cdot \left(C'_{1,s} \cdot \varepsilon + C'_{2,s} \cdot \frac{\|q - q_s\|_1}{\sqrt{s}} \right) \\ &\leq C_{1,s} \cdot \varepsilon + C_{2,s} \cdot \frac{\|q - q_s\|_1}{\sqrt{s}}, \end{aligned} \quad (4.18)$$

where the second inequality expression can be proved based on [71]. When a specific input sequence is chosen, the 2-norm of $S_{\text{order}}^{-1} \Psi^{-1}$ can also be calculated. ■

Note that the general formulation of compressive sensing algorithm requires that the transformation matrix Ψ has a dimension of $N \times N$. In order to utilize the compressive sensing framework to identify the Preisach operator using Eq. (4.15), the dimension of the matrix S needs to be $N \times N$, which requires that the initial input sequence has N entries as well. For input sequence with a different number of entries, or that with N entries but the corresponding matrix S is not invertable,

Eq. (4.15) cannot be adopted directly. Moore-Penrose pseudoinverse [81] could potentially be utilized as an approximation of the inverse of S . This work considers cases that the input sequence has N entries and the corresponding S is invertable.

4.3 Simulation Results

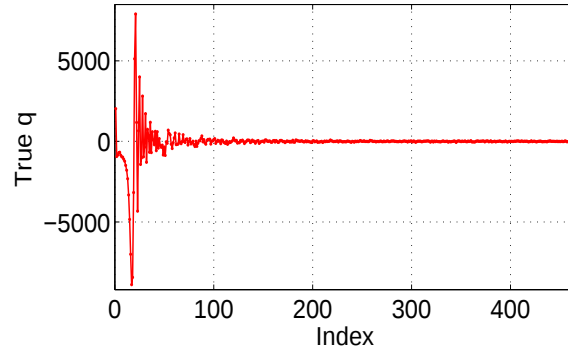
Consider a Preisach operator with a zero bias μ_0 , and discretization level $L = 30$. Each of the 465 density values is generated following a uniform distribution on the interval $[0,12]$. Apply the “damped oscillation” input sequence and obtain the corresponding output. The measured output is simulated with added noise that follows a uniform distribution on the interval $[-5,5]$, which is about 0.23% of the largest output value. In order to quantitatively examine the relationship between the reconstruction performance and the number of measurements, the compressive sensing-based identification approach is compared with a constrained least-squares scheme. The constrained least-squares method is realized with the Matlab command *lsqnonneg* to identify the vector of parameters that meets the sign constraints [25]. For each number of measurements used in the identification, simulation is run 1000 times (thus with different sets of density values and different sets of chosen input-output data), and the performance is averaged among all of the results.

Fig. 4.3(a) shows a typical example of real q . It is seen that the dominant elements only cover the low frequencies. The largest absolute value of elements of q is 8,665.8, and it is found that as many as 363 elements are less than 1% of the largest elements of q . Fig. 4.3(b) shows a typical example of real q_{order} . It is seen that the dominant elements also cover the low frequencies. The largest absolute value of elements of q is 18,703.4, and it is found that as many as 415 elements are less than 1% of the largest elements of q . It is verified that q_{order} is more approximately sparse than q . It is anticipated and also verified that “CS Order” would achieve better model reconstruction and

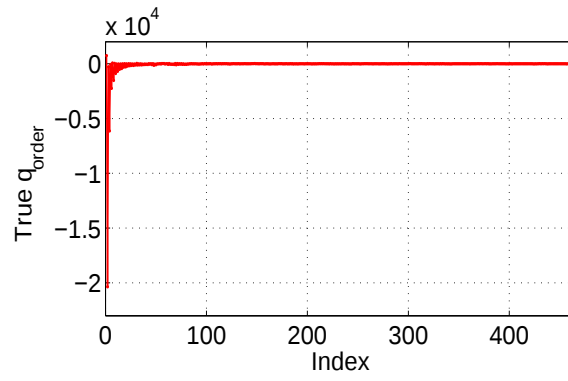
estimation performances than “CS”. Fig. 4.3(c) shows the reconstruction performance comparison of q between “CS” and “CS Order”. It is evident that the “CS Order” would achieve much better reconstruction performance of q_{order} . On average, the RMSE error of q_{order} based on “CS Order” has about 17.3% of the RMSE error of q using “CS”.

Fig. 4.4(a) shows the reconstruction performance comparison of density w between “Least Squares”, “CS Without DCT”, “CS”, “CS Order”, and “CS Order Non-negative”, where “CS Without DCT” directly uses the compressive sensing algorithm to identify the density w based on Eq. (4.12). It is shown that except the “CS Without DCT”, all of the CS-related reconstruction approaches result in better density reconstruction performances than that of the “Least Squares”. Among these CS-related approaches, “CS” is the worst, and “CS Order Non-negative” is consistently the best. On average, the RMSE errors of reconstructed density based on “CS Order Non-negative”, “CS Order”, and “CS” are 28.1%, 34.5%, and 73.1% of the error using “Least Squares”, respectively. The “CS Without DCT” cannot faithfully reconstruct the density due to the fact that the density vector is not approximately sparse, and the measurement matrix does not follow the RIP condition, and is not considered in the following of the manuscript. Fig. 4.4(b) shows the normalized modeling error comparison. The error is obtained as follows: first, calculate the RMSE between the output of the identified Preisach operator and that of the actual output, then divide the RMSE by the output range. Normalization of the RMSE facilitates the assessment of the algorithm performance. It is shown that “CS Order Non-negative” consistently produces the smallest modeling error, followed by “CS Order”, “CS”, and “Least Squares”. On average, the RMSE errors based on “CS Order Non-negative”, “CS Order”, and “CS” are 18.3%, 22.6%, and 47.2% of the error using “Least Squares”, respectively.

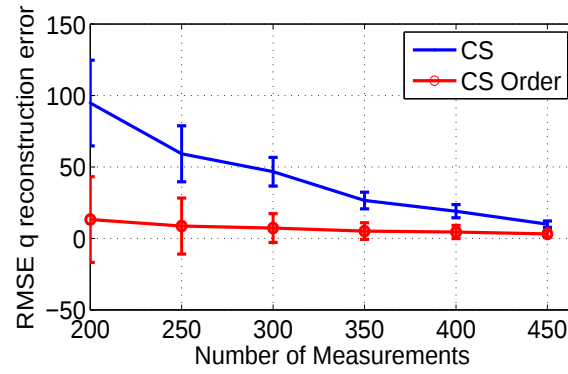
Based on *Proposition 3*, the CS-related approaches have bounded reconstruction error under noisy measurements. It is shown that when the magnitude of the measurement noise increases, the



(a)



(b)



(c)

Figure 4.3: (a) Signal q showing the sparseness; (b) signal q_{order} is more approximately sparse than q ; (c) the reconstruction performance comparison based on “CS” and “CS Order”.

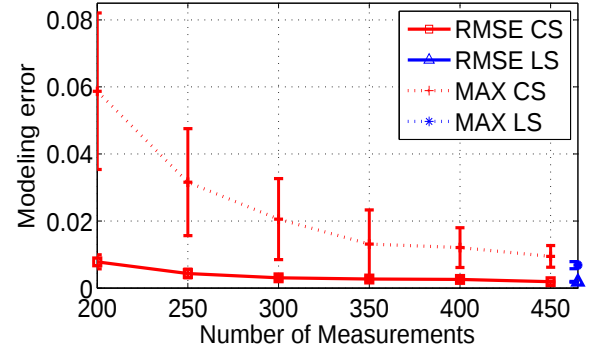
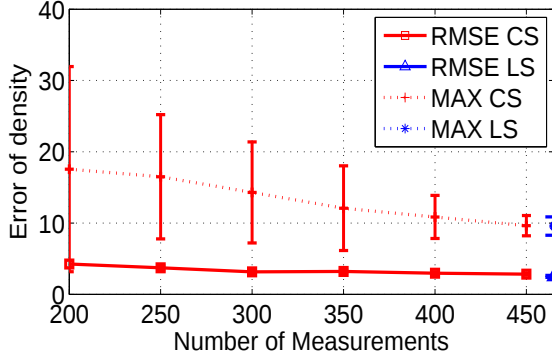


Figure 4.4: (a) Density w reconstruction error comparison; (b) the modeling error comparison.

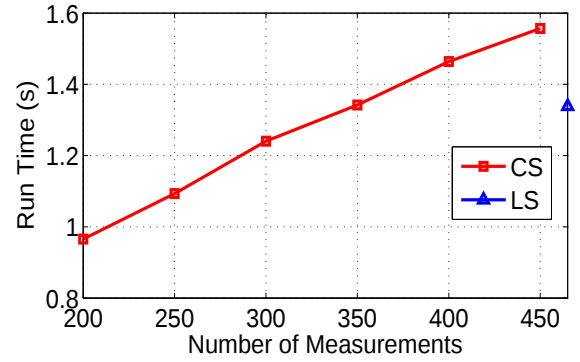
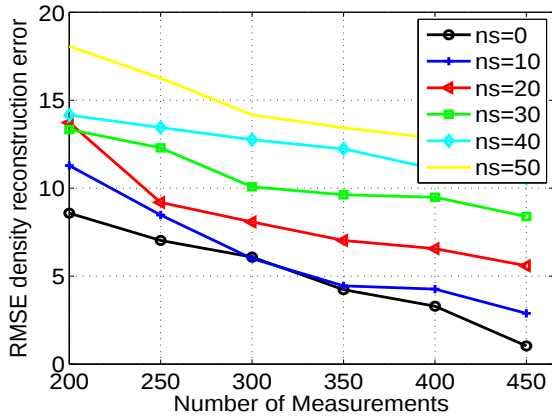


Figure 4.5: (a) Density reconstruction error with varying measurement noise based on “CS”; (b) the average identification run-time comparison.

magnitude of the reconstruction error becomes larger. Denote the measurement noise follows a uniform distribution on the interval $[-\frac{ns}{2}, \frac{ns}{2}]$, Fig. 4.5(a) shows the corresponding RMSE density reconstruction error with varying amplitudes of measurement noise. Fig. 4.5(b) shows the average run-time between the CS-related approaches and the least-squares algorithm. The computations are run in Matlab on a computer with Intel(R) Core(TM) i7-2600 3.40 GHz CPU and 4.00 GB memory. It is seen that when the number of measurements is increasing, “CS” and “CS Order” are much more efficient than “Least Squares”. The ordering of the output consumes much less time than the generic path-following primal-dual method to solve the compressive sensing-based reconstruction algorithm. “CS Order Non-negative” is slightly more time-consuming than the “Least Squares” approach, while it can ensure the identified density function contains no negative elements. On average, the average identification run-time of “Least Squares”, “CS”, “CS Order”, and “CS Order Non-negative” are 1.18s, 0.28s, 0.29s, and 1.99s, respectively.

To further validate the proposed approach, a random input sequence shown in Fig. 4.6(a) is used, Fig. 4.6(b) shows the corresponding output corrupted with a noise that has the aforementioned distribution. Fig. 4.6(c) shows the normalized model estimation error comparison under the random input. It is shown that “CS Order Non-negative” produces the smallest modeling error, followed by “CS Order”, “CS”, and “Least Squares”. On average, the RMSE errors of reconstructed density based on “CS Order Non-negative”, “CS Order”, and “CS” are 22.3%, 24.4%, and 48.8% of the error using “Least Squares”, respectively. For example, when the number of measurements is 300, the average RMSE errors using “Least Squares”, “CS”, “CS Order”, and “CS Order Non-negative” are 0.048, 0.024, 0.011, and 0.010, respectively.

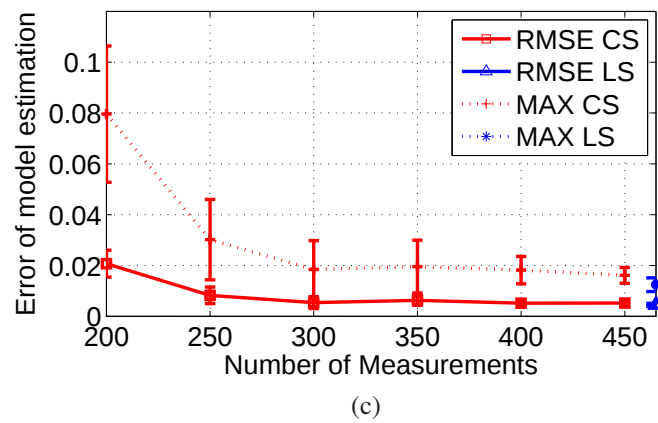
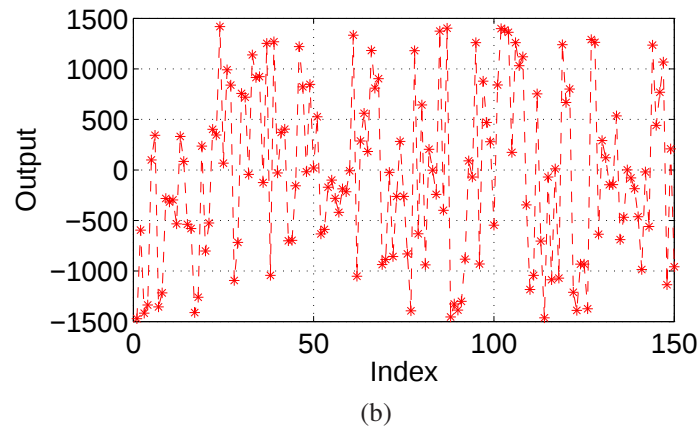
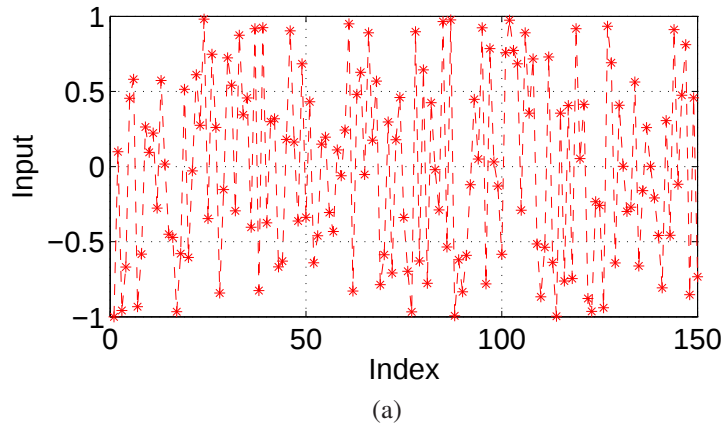


Figure 4.6: (a) A random input sequence for model validation; (b) corresponding output under the random input sequence in (a); (c) model estimation error comparison.

4.4 Experimental Results

VO₂ is an interesting class of smart materials with a myriad of microactuation, optical, and memory applications. It undergoes an insulator-to-metal transition (IMT) at around 68 °C, during which resistance [82], induced mechanical stress [26], and optical transmittance demonstrate pronounced hysteresis. The proposed identification algorithm is verified in experiments for identifying and characterizing the hysteresis between the voltage input and the deflection output of a VO₂-integrated microactuator.

4.4.1 Measurement Setup

The experimental setup used is shown in Fig. 4.7(a). The microactuator used in this setup consisted of a silicon dioxide (SiO₂) microcantilever with patterned VO₂ film inside the structure and a patterned metal layer (Au/Ti) on top. The VO₂ film was used as the active actuation element in the cantilever, while the metal layer was used to form the heating element and the traces for the VO₂ resistance contacts. The measurement system was based on a laser scattering technique, using an IR laser ($\lambda=808$ nm) and a position sensitive detector (PSD) to track the displacement of the cantilever. A charge couple device (CCD) camera was used for alignment and calibration purposes. A data acquisition system and field programmable gate array (DAQ/FPGA) with a computer interface was used to automate the control/monitor of electric signals. The power of the sensing laser (222 mW) was calibrated to be the minimum possible to be sensed by the PSD without heating the cantilever due to photon absorption. The voltage output (VD) of the PSD was linearly proportional to the position of the laser. Using images captured by the CCD camera this voltage (VD) was mapped to the deflection of the cantilever. The chip containing the microactuator was inside a side braze packaging (wire-bonded), which was connected to the DAQ/FPGA. The current I_H shown

in Fig. 4.7(b) was used to control the cantilever's temperature by Joule heating. The current was generated using two resistances in series: the heater resistance and an external resistance, whose only purpose was to limit the maximum current (12.78 mA) that can be applied to the system. An input voltage from the DAQ/FPGA and the computer interface was used to generate this current.

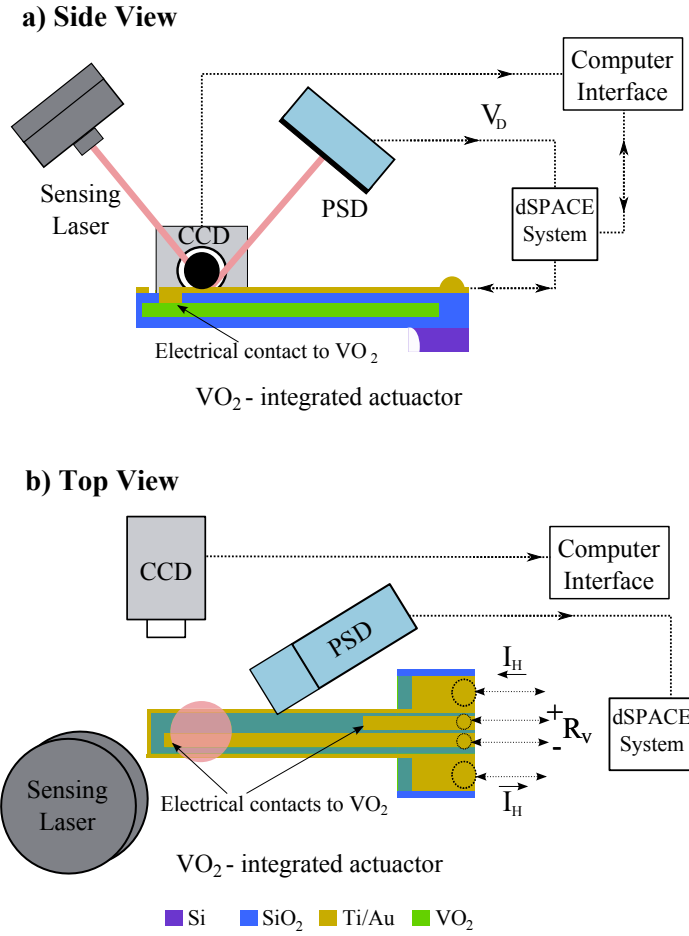


Figure 4.7: a) Side view schematic for the measurements setup for deflection of a microactuator with an integrated heater; b) Top view of the VO_2 -based integrated actuator devices.

4.4.2 Identification and Verification

A VO_2 -integrated silicon microactuator is subject to two actuation effects when its temperature is varied [26]. The first is the phase transition-induced stress, which makes the beam bend towards the VO_2 layer when the microactuator is heated. The deflection due to phase transition can be

modeled by a Preisach operator. The second effect is the differential thermal expansion-induced stress, which makes the beam bending away from the VO₂ layer under heating. The latter effect is modeled with a linear term. As a result, the hysteresis between the deflection and the temperature is non-monotonic, and can be modeled as

$$\tilde{u}(n) = \mu_0 + c_d v(n) + \sum_{i=1}^L \sum_{j=1}^{L+1-i} \mu_{ij} s_{ij}(n). \quad (4.19)$$

The model expressed in Eq. (4.19) can still be expressed similarly as Eq. (4.3) assuming the discretization level $L = 30$, where w contains $N = L(L+1)/2 + 2 = 467$ elements, and can be written as $w = \left(w_1 \ w_2 \ \cdots \ w_{L(L+1)/2} \ \mu_0 \ c_d \right)^\top$. Apply the following input sequence to the system: the first 466 elements are the same as a damped oscillation sequence shown in Fig. 4.2, the 467th element of the input can be any value other than 0 to ensure that the corresponding matrix S is invertible. S is a 467×467 matrix, where $S(n, k) = s_{ij}(n)$, $S(n, L(L+1)/2 + 1) = 1$, and $S(n, L(L+1)/2 + 2) = v[n]$. Fig. 4.8(a) shows the non-monotonic hysteresis behavior between the voltage input and the deflection output. Fig. 4.8(b) shows the corresponding density function (true density) identified based on the 467 measurements shown in Fig. 4.8(a).

It can be proved that by applying the aforementioned input sequence, the rank of the corresponding S is 467. Instead of utilizing all of the 467 corresponding output deflection measurements, a part of the output measurements were randomly chosen for identification. For each number of measurements used in the compressive sensing algorithms and least-squares scheme, reconstruction algorithms are run 1000 times (thus with different sets of chosen input-output data), and the performance is averaged among all of the simulations.

When the number of measurements used for identification is 300, Fig. 4.9(a)-(d) show typical density function reconstruction error performances. It is calculated that the RMSE error using

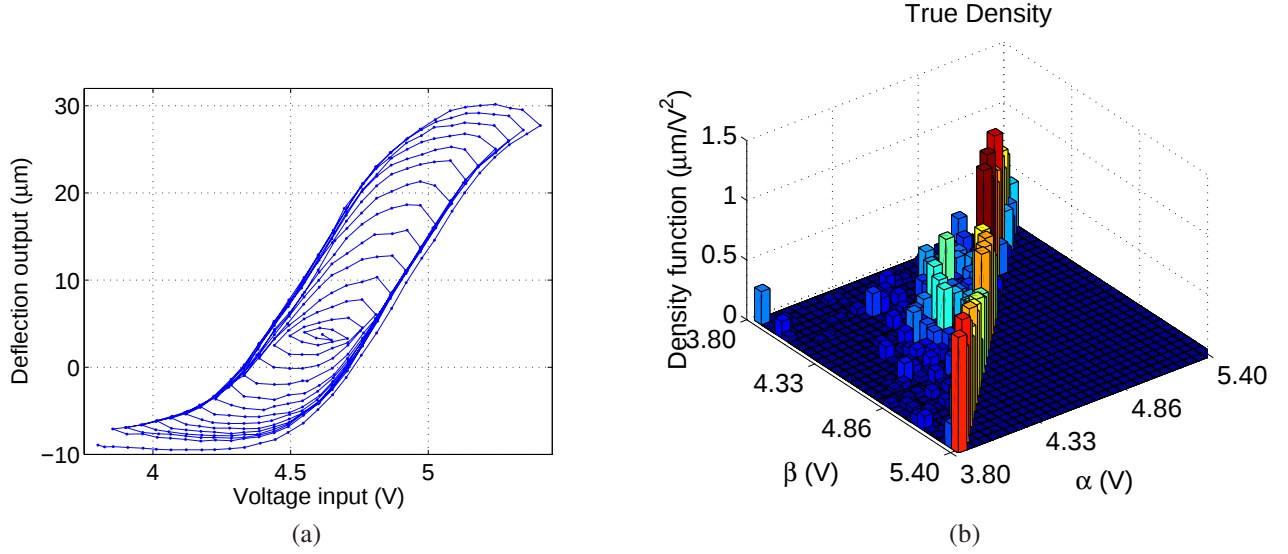


Figure 4.8: (a) Input-output data for identifying Preisach operator ($L = 30$); (b) true density function identified based on 467 measurements.

“Least Squares”, “CS”, “CS Order”, and “CS Order Non-negative” are $0.122\mu\text{m}/\text{V}^2$, $0.116\mu\text{m}/\text{V}^2$, $0.096\mu\text{m}/\text{V}^2$, and $0.059\mu\text{m}/\text{V}^2$, respectively.

Fig. 4.10 shows the normalized modeling error comparison. It is shown that “CS Order Non-negative” consistently produces the smallest modeling error, followed by “CS Order”, “CS”, and “Least Squares”. On average, the RMSE errors based on “CS Order Non-negative”, “CS Order”, and “CS” are 13.4%, 25.3%, and 76.8% of the error using “Least Squares”, respectively.

The effectiveness of the compressive sensing-based identification is further examined by comparing the model estimation performance under a random input shown in Fig. 4.11(a). The measured deflection output is shown in Fig. 4.11(b). Fig. 4.11(c) shows the model estimation errors under the models identified with the compressive sensing schemes and the least-squares scheme, respectively. On average, the RMSE errors of reconstructed density based on “CS Order Non-negative”, “CS Order”, and “CS” are 72.9%, 78.4%, and 84.5% of the error using “Least Squares”, respectively. For example, when the number of measurements used for identification is 300, the average RMSE errors using “Least Squares”, “CS”, “CS Order”, and “CS Order Non-negative” are

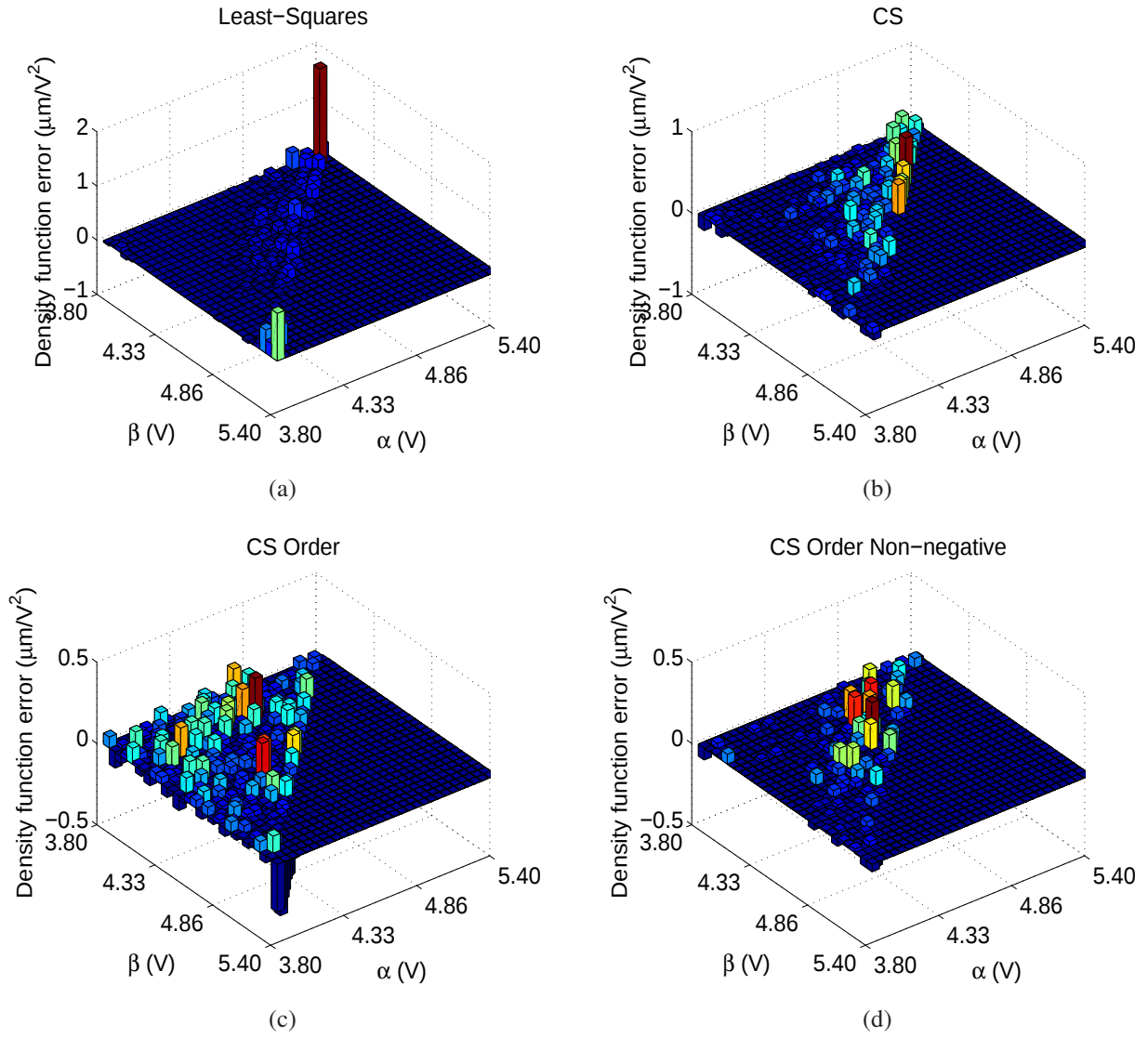


Figure 4.9: Density reconstruction error comparison based on (a) Least Squares; (b) CS; (c) CS Order; (d) CS Order Non-negative approaches.

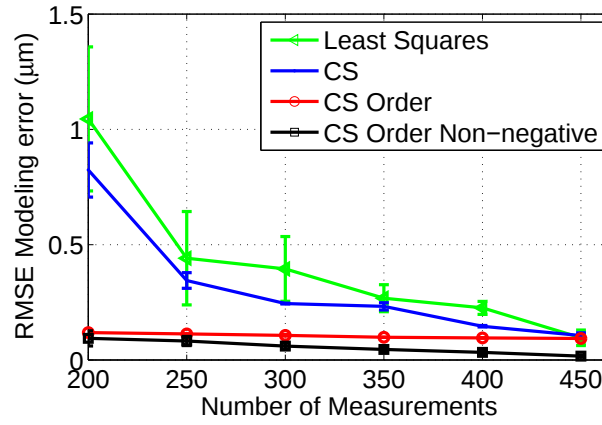
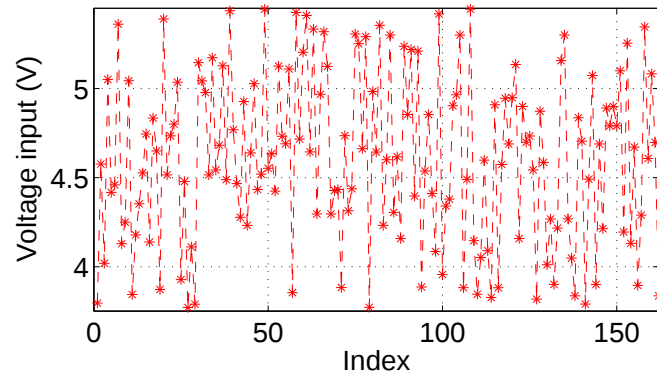
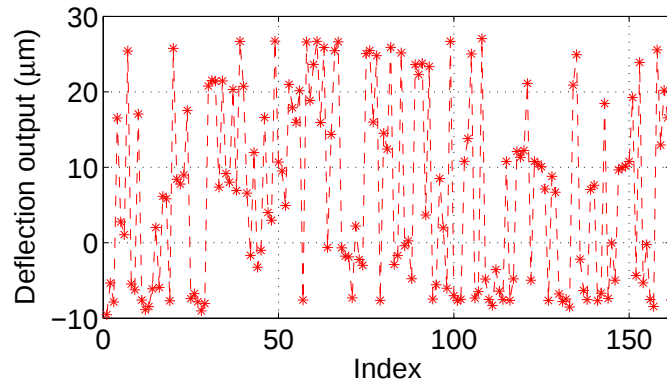


Figure 4.10: The average RMSE modeling error comparison.

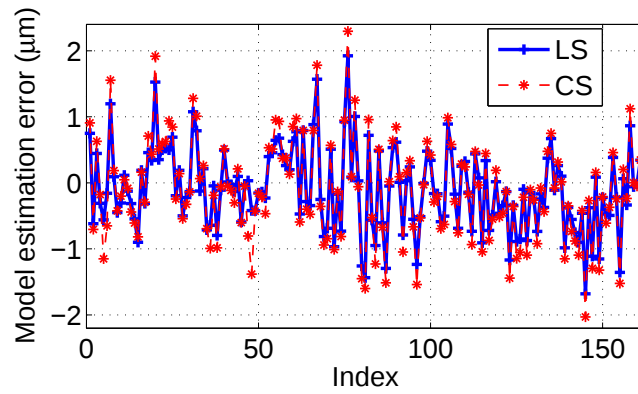
1.010 μm, 0.891 μm, 0.831 μm, and 0.778 μm, respectively.



(a)



(b)



(c)

Figure 4.11: Density reconstruction error based on (a) a random input sequence for model validation; (b) output of the random input sequence in (a); (c) model estimation errors.

Chapter 5

Modeling and Inverse Compensation of Non-monotonic Hysteresis based on the Preisach Operator

In this chapter, the systematic studies on the modeling and inverse compensation of non-monotonic hysteresis exhibited by VO_2 -coated microactuators are presented. First, a physics-motivated model that accounts for the two (opposite) actuation mechanisms is presented. The first mechanism is the stress resulting from structural changes in VO_2 , which is modeled with a monotonic Preisach operator. The second mechanism is the differential thermal expansion effect. Since the thermal expansion coefficient (TEC) of VO_2 depends on the phase mixture of the material, a linear function of the temperature is taken to efficiently model the phase fraction of VO_2 , which results in a quadratic operator for the thermal expansion-induced actuation. The parameters of the model are identified. Second, an efficient inverse compensation scheme is developed for the proposed non-monotonic hysteresis model by adapting the scheme used in [9] for a Preisach operator with nonnegative, piecewise constant density function. The effectiveness of the model and the inverse compensation scheme is demonstrated in experiments, with comparison to two other approaches, one based on a Preisach operator with a signed density function and the other based on a polynomial model.

5.1 Experimental Characterization of VO₂-coated Microactuators

5.1.1 Material Preparation and Experimental Setup

A 172 nm thick VO₂ layer was deposited by pulsed laser deposition on a 300 μm long silicon cantilever (MikroMasch CSC12) with width and thickness of 35 μm and 1 μm , respectively. The deposition was conducted inside a vacuum chamber. The deposition followed a similar procedure as in previous experiments [36], where a krypton fluoride excimer laser (Lambda Physik LPX 200, $\lambda = 248 \text{ nm}$) was focused on a rotating metallic vanadium target with a 10 Hz repetition rate. The background pressure was 10^{-6} Torr and throughout the deposition was kept at 20 mTorr with gas flows of 10 (argon) and 15 (oxygen) standard cubic centimeter per minute (sccm).

Fig. 1.1 shows two superimposed scanning electron microscopy (SEM) pictures of the prepared VO₂-coated cantilever, when the substrate temperature was 30 °C and 90 °C, respectively. VO₂ is in pure M₁ and R phases at those two temperatures. A total tip displacement change of about 70 μm is observed in Fig. 1.1, illustrating the large bending the microactuator is capable of generating. The considerable amount of initial curvature at room temperature is due to the residual stress after deposition. Since the change of curvature is of more interest, the curvature change from the initial curvature is taken as the output.

In order to experimentally measure the tip deflection as a function of temperature, the setup shown in Fig. 5.1 was used, which was similar to the one used before [36]. Here, the micro-cantilever was glued with a highly thermal conductive silver paint to a glass substrate that was directly in contact with a Peltier heater. The heater was controlled in closed loop with a commercially available benchtop temperature controller (Thorlabs, TED-4015) connected to a temperature

sensor (AD592), with a precision in temperature control of ± 0.1 °C. A custom-made current controller circuit was used to power an infrared laser ($\lambda = 808$ nm) with a maximum power of 20 mW. The laser spot was focused on the tip of the cantilever and the reflected laser light was captured by a one-dimensional position sensitive detector (PSD) (Hamamatsu S3270). The PSD outputs a voltage proportional to the position of the reflected laser spot on its active area. Bending of the VO₂-coated cantilever produces an angular displacement on the reflected laser spot, which changes the output voltage of the PSD. The laser intensity was kept at the lowest detectable by the PSD, in order to obtain good signal-to-noise ratio while minimizing heating of the cantilever by the laser. The output voltage from the PSD was measured with an analog input module (NI 9201), which was attached to an embedded real-time controller (NI cRIO 9075), and a LabView program was created in order to automate the deflection measurements.

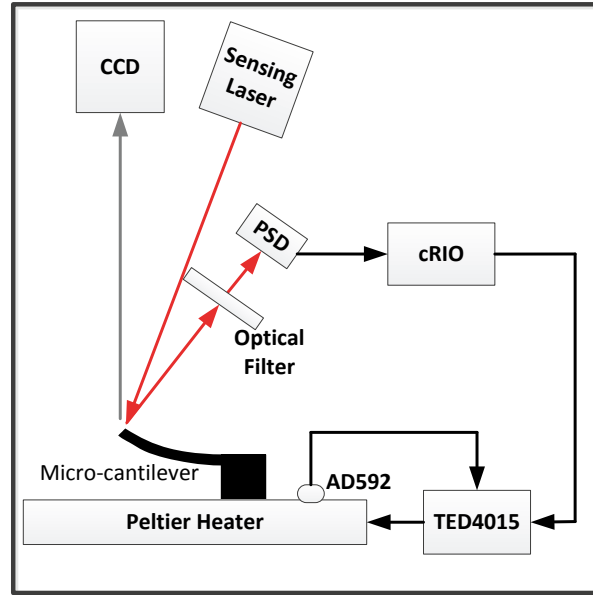


Figure 5.1: Setup used for measuring the cantilever tip deflection as a function of temperature.

The measured tip deflection Δz was converted to the curvature κ using the geometry illustrated in Fig. 5.2. The radius of curvature, $r = 1/\kappa$, is related to Δz via:

$$\Delta z = \sin\left(\frac{\delta}{2}\right) \cdot AB = 2r \cdot \sin^2\left(\frac{L}{2r}\right), \quad (5.1)$$

where $\delta = \frac{L}{r}$. For the microactuator studied in this work, the small angle approximation typically used ($\Delta z \approx \frac{L^2}{2r}$) will not be valid because of the large bending, and the transcendental equation (5.1) is numerically solved for the curvature.

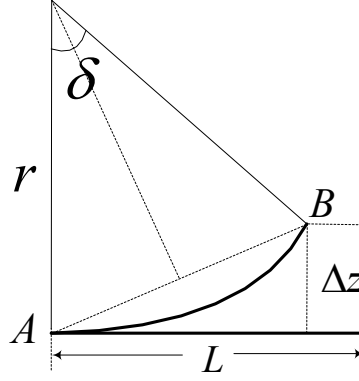


Figure 5.2: Illustration of the geometric relationship between the curvature and the tip deflection of a bent cantilever.

5.1.2 Characterization of Non-monotonic Hysteresis

A creep test was conducted and no obvious creep was found. The temperature was varied by a step and then until constant, while the deflection was measured for the whole process. As Fig. 5.3 shows, the deflection under unchanged temperature values varied from $[67.378, 67.667] \mu\text{m}$ and $[15.082, 15.164] \mu\text{m}$, respectively. These minute variations are mainly attributed to error in temperature control (accuracy $\pm 0.1^\circ\text{C}$), hence creep is not considered.

A set of experiments were conducted to obtain the curvature of the VO_2 microactuator as a function of temperature. The temperature range was chosen to be from 21°C to 84°C , to fully cover the phase transition regime. The temperature profile in time followed a pattern of damped oscillations (not shown), to provide sufficiently rich excitation for the identification of the Preisach

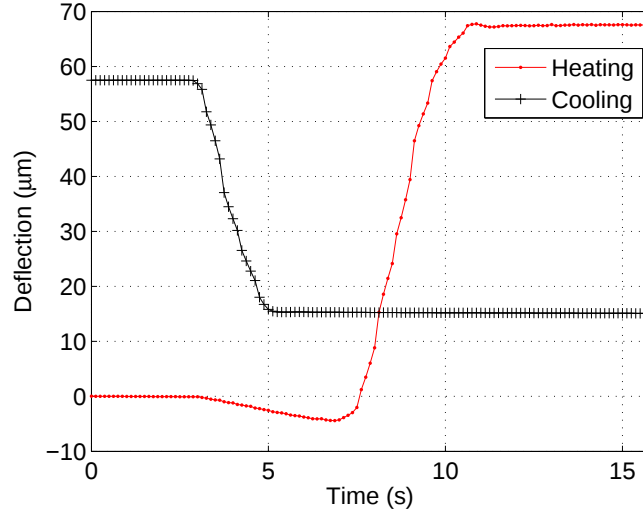


Figure 5.3: Deflection as a function of time through heating and cooling temperature steps. There is no observable creep.

hysteresis model. Fig. 5.4(a) shows the measured nested hysteresis loops between the actuator curvature and temperature. Notice that the apparent phase transition temperature is shifted from the typical value of 68 °C. This is attributed to the heating effect by the deflection-measuring laser; recall that the recorded temperature was only for the Peltier heater located underneath the sample. In this work, the heat contribution from the measurement laser is considered to be constant (which is a reasonable assumption), and thus the temperature of the Peltier heater is taken as the input.

Non-monotonic hysteresis can be clearly observed in Fig. 5.4(a). As the temperature is increased, the curvature first decreases slightly, then increases abruptly, and finally decreases slightly again when the temperature is sufficiently high. An analogous trend holds true when the temperature is decreased. The non-monotonic behavior can be explained by two competing actuation mechanisms. On one hand, changes in the crystalline structures during the $M_1 \rightarrow R$ phase transition result in microcantilever bending toward the VO_2 layer. Vanadium ions are reordered during the phase transition, where one unit cell in the M_1 phase corresponds to two unit cells in the R phase. The crystalline plane parallel to the substrate changes from $(011)_{M_1}$ in the M_1 phase to

$(110)_R$ in the R phase. From the lattice parameters [83], it is calculated that the crystallographic plane of VO_2 that is parallel to the cantilever surface for this case ($(011)_M$) decreases its area by 1.7% (on heating), which generates a strain of approximately -0.083 [36], causing a drastic bending towards the VO_2 layer side. On the other hand, since the thermal expansion coefficient (TEC) of VO_2 for both the M_1 phase and the R phase [83] are larger than that of silicon [84], the differential thermal expansion-induced stresses result in an opposite bending effect from that of the phase transition effect.

There are several additional interesting observations from Fig. 5.4(a). First, the curvature-temperature relationship is hysteretic only in the intermediate temperature regime, and becomes a single-valued function at both the low and high temperature ends. This provides support for the two actuation effects discussed earlier; phase transition dominates the intermediate temperature region, while at the low or high temperature ends, VO_2 is in a single phase (M_1 or R phase, respectively), and the differential thermal expansion takes dominance. Second, the slope of curvature versus temperature at the low-temperature end is different from that at the high-temperature end, suggesting that the TEC of VO_2 changes with the material phase. This is consistent with what can be found in literature [83].

Extensive experiments are conducted to characterize the repeatability of the actuation behavior, and the hysteresis loops measured on different days (with the same temperature input sequence) are found to be nearly identical (shown in Fig. 1.2).

5.2 Non-monotonic Hysteresis Model

5.2.1 Actuation Effect due to Phase Transition

The proposed model for the curvature output κ of the VO₂-coated cantilever comprises the contribution κ_P due to the phase transition and the contribution κ_E due to the differential thermal expansion. The phase transition contribution is monotonically hysteretic with respect to the temperature T , thus will be modeled by a Preisach operator [27, 85] with non-negative density function μ :

$$\begin{aligned}\kappa_P(t) &= \Gamma[T(\cdot); \zeta_0](t) \\ &= c_0 + \int_{\mathcal{P}_0} \mu(\beta, \alpha) \gamma_{\beta, \alpha}[T(\cdot); \zeta_0(\beta, \alpha)](t) d\beta d\alpha,\end{aligned}\tag{5.2}$$

here c_0 is some constant bias, $T(\cdot)$ denotes the temperature history, $T(\tau)$, $0 \leq \tau \leq t$, $\mathcal{P}_0$ is called the Preisach plane $\mathcal{P}_0 \triangleq \{(\beta, \alpha) : T_{\min} \leq \beta \leq \alpha \leq T_{\max}\}$, where $[T_{\min}, T_{\max}]$ denotes the temperature range for phase transition, and finally, $\gamma_{\beta, \alpha}$ denotes the basic hysteretic unit (hysteron): for a pair of thresholds (β, α) and an initial condition $\zeta_0(\beta, \alpha) \in \{-1, 1\}$, the output of the hysteron is defined as:

$$u(t) = \gamma_{\beta, \alpha}[T(\cdot); \zeta_0(\beta, \alpha)] = \begin{cases} +1 & \text{if } T(t) > \alpha \\ -1 & \text{if } T(t) < \beta \\ u(t^-) & \text{if } \beta \leq T(t) \leq \alpha \end{cases}, \tag{5.3}$$

where $T(\cdot)$ is the temperature input history $T(\tau)$, $0 \leq \tau \leq t$, and $u(t^-) = \lim_{\varepsilon > 0, \varepsilon \rightarrow 0} u(t - \varepsilon)$.

Note that $\mathcal{P}_0$ can be divided into two regions according to the outputs of hysterons, and the boundary of the two regions (called memory curve and denoted ψ) represents equivalently the state

and thus determines the output of the Preisach operator. For this reason, the initial state function ζ_0 can be replaced by an initial memory curve in the Preisach plane.

5.2.2 Differential Thermal Expansion Effect

When the temperature increases (decreases, resp.), a material typically expands (shrinks, resp.). For a two-layer beam, the difference in the thermal expansion of individual layers results in bending. The VO₂-coated silicon cantilever curvature κ_E due to differential thermal expansion at a temperature T can be derived following standard analysis [86]:

$$\kappa_E = -\frac{6(1+m)^2(C_{\text{VO}_2} - C_{\text{Si}})(T - T_0)}{h \left(3(1+m)^2 + (1+mw)(m^2 + \frac{1}{mw}) \right)}, \quad (5.4)$$

where h is the total thickness of the beam, m is the ratio of the VO₂ layer thickness to that of the silicon layer, w is the ratio of the modulus of elasticity of the VO₂ layer to that of the silicon layer, C_{VO_2} and C_{Si} are the TECs of the VO₂ and silicon, respectively, and T_0 is the room temperature (20 °C). In (5.4), it is defined that κ_E is positive when the beam bends toward the VO₂ layer.

As mentioned earlier, the TEC of VO₂ in the M₁ phase is different from that in the R phase. Since the phase transition spans through the temperature range $[T_{\min}, T_{\max}]$, both phases coexist within that temperature range. If C_{M_1} and C_{R} are the TECs of VO₂ in the M₁ and R phases, respectively, and $\theta(T)$ is the material fraction of the R phase at a particular temperature T , then the effective TEC of VO₂ during the phase transition can be represented with respect to T as:

$$C_{\text{VO}_2} = (1 - \theta(T))C_{\text{M}_1} + \theta(T)C_{\text{R}}. \quad (5.5)$$

In general, the R phase fraction θ is hysteretic with respect to the temperature T . To make the problem tractable, θ is approximated by a linear function of T , which is supported by experimental

observations [87]:

$$\theta = \begin{cases} 0, & \text{if } T < T_{\min} \\ \frac{T - T_{\min}}{T_{\max} - T_{\min}}, & \text{if } T_{\min} \leq T \leq T_{\max} \\ 1, & \text{if } T > T_{\max}. \end{cases} \quad (5.6)$$

Combining (5.4) and (5.5), we rewrite κ_E as:

$$\kappa_E = -(k_0(1 - \theta(T)) + k_1\theta(T))(T - T_0), \quad (5.7)$$

where $k_0 = \frac{6(1+m)^2(C_{M1} - C_{Si})}{h(3(1+m)^2 + (1+mw)(m^2 + \frac{1}{mw}))}$ and $k_1 = \frac{6(1+m)^2(C_R - C_{Si})}{h(3(1+m)^2 + (1+mw)(m^2 + \frac{1}{mw}))}$. Since $C_{M1} < C_R$ [83], $k_1 > k_0$. With the approximation (5.6) for $\theta(T)$, the differential thermal expansion-induced curvature has a quadratic dependence on T .

By adding (5.2) and (5.7), we obtain the total curvature with a new hysteresis operator Ω :

$$\begin{aligned} \kappa(t) &= \kappa_P(t) + \kappa_E(t) = \Omega[T(\cdot); \zeta_0](t) \\ &\stackrel{\Delta}{=} c_0 + \Gamma[T(\cdot); \zeta_0](t) - (k_0(1 - \theta(T(t))))(T(t) - T_0) \\ &\quad + k_1\theta(T(t))(T(t) - T_0). \end{aligned} \quad (5.8)$$

Note that for Ω , only the contribution κ_P is memory-dependent, so Ω shares the same state (or memory curve) as Γ . In particular, they will share the same initial memory curve.

5.3 Model Identification and Validation

5.3.1 Parameter Identification

The parameters of the model (5.8) include the density function μ of the Preisach operator, and the constants c_p , k_0 , and k_1 . For the identification of a Preisach density function, a discretization step is typically involved. The discretization scheme adopted in this work approximates the density by a piecewise constant function, where the density value is constant within each lattice cell but varies from cell to cell [6].

The input range is discretized uniformly into M levels, which results in $M(M+1)/2$ cells, leading to a total of $K = M(M+1)/2$ weight parameters. The actual operating range for the input, $[T'_{\min}, T'_{\max}]$, is considered larger than the phase transition region $[T_{\min}, T_{\max}]$. In other words, $T'_{\min} < T_{\min} < T_{\max} < T'_{\max}$.

In a discrete time setting, the contribution $\kappa_P[n]$ at time n is:

$$\kappa_P[n] = c_p + \sum_{i=1}^M \sum_{j=1}^{M+1-i} \mu_{ij} s_{ij}[n], \quad (5.9)$$

where $s_{ij}[n]$ represents the *signed* area of cell (i, j) , μ_{ij} represents the density of cell (i, j) . Note that the signed area of each cell is defined as the area occupied by hysterons with output $+1$ minus that occupied by hysterons with output -1 . For ease of presentation, the cells (i, j) are ordered with a single index $l = 1, 2, \dots, K$, and the density and signed area of the l -th cell are denoted (with abuse of notation) by μ_l and s_l , respectively. Eq. (5.9) is rewritten as:

$$\kappa_P[n] = c_0 + \sum_{l=1}^K \mu_l s_l[n]. \quad (5.10)$$

When the input $T[n]$ is in $[T'_{\min}, T_{\min}]$, all hysterons attain the value of -1 and the Preisach

operator is at the negative saturation, $\kappa_P[n] = c_0 - \kappa_0$, where $\kappa_0 = \sum_{l=1}^K \mu_l$. Similarly, positive saturation is reached when the temperature $T[n]$ is within $[T_{\max}, T'_{\max}]$.

On the other hand, the contribution $\kappa_E[n]$ is:

$$\kappa_E[n] = -(k_0(1 - \theta(T[n])) + k_1\theta(T[n]))(T[n] - T_0). \quad (5.11)$$

Combining (5.10) and (5.11), we obtain the total curvature output as:

$$\kappa[n] = c_p + \sum_{l=1}^K \mu_l s_l[n] - (k_0(1 - \theta(T[n])) + k_1\theta(T[n]))(T[n] - T_0). \quad (5.12)$$

Preisach density functions are assumed to be nonnegative, $\mu_l \geq 0$. In addition, k_0 and k_1 are positive from their physical meanings. Finally, $c_p > 0$ since VO₂-coated microcantilevers have positive curvature bias. A constrained least-squares method, realized with the Matlab command *lsqnonneg*, is utilized to identify the vector of parameters, $[\mu_1 \ \mu_2 \ \mu_3 \ \cdots \ \mu_K \ c_p \ k_0 \ k_1]^T$, that meets the sign constraints.

5.3.2 Experimental Results

To effectively identify the model parameters, the input needs to provide sufficient excitation for all cells of the Preisach operator. One type of such input sequences takes the form of damped oscillations, which produces nested hysteresis loops and is adopted in this work. Based on empirical knowledge, the temperature of the closed-loop-controlled Peltier device can settle around a set temperature within about 3 s. A wait time of 8 s was chosen between temperature setpoints to ensure that the thermal steady state has been reached. While the experiment will remain to be quasi-static if the waiting time is longer than 8 s, it is not advisable to make it much shorter.

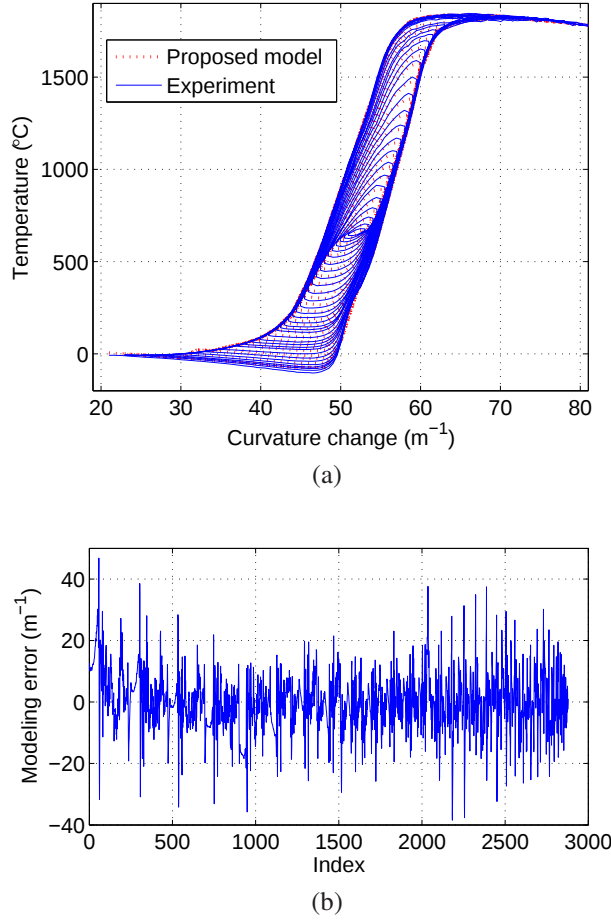


Figure 5.4: Proposed model. (a): Measured non-monotonic curvature-temperature hysteresis and that based on the proposed model. (b): Modeling error for the entire temperature sequence.

The full range of temperature input $[T'_{\min}, T'_{\max}]$ is $[21, 84]$ °C. From Fig. 5.4(a), the phase transition region $[T_{\min}, T_{\max}]$ is determined to be $[30, 70]$ °C. The level of discretization M for the Preisach plane is chosen to be 20. Fig. 5.4(a) compares the measured hysteresis loops and those based on the identified model, and Fig. 5.4(b) shows the corresponding modeling error for the entire temperature sequence, which is mostly bounded by 30 m^{-1} , compared with the total curvature change range $[-104, 1846] \text{ m}^{-1}$. Fig. 5.5 shows the identified density function of the Preisach operator. c_p , k_0 , and k_1 have been identified to be 1026.7 m^{-1} , $2.8 \text{ m}^{-1}\text{K}^{-1}$, and $4.3 \text{ m}^{-1}\text{K}^{-1}$, respectively.

For comparison purposes, two additional models for the non-monotonic hysteresis are consid-

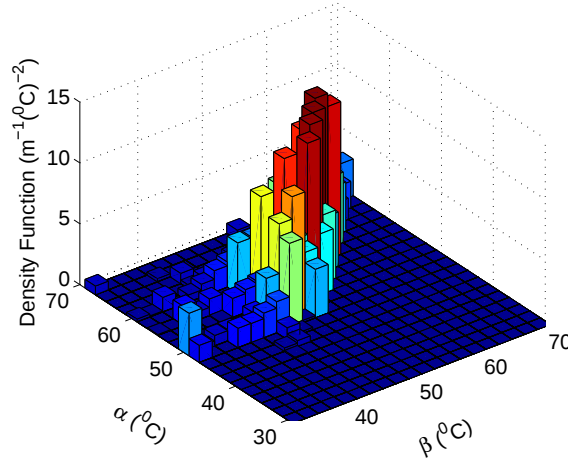


Figure 5.5: Identified Preisach density function for the proposed model.

ered: a Preisach operator with a signed density function, and a nonlinear single-valued function represented by a high-degree polynomial.

The signed Preisach operator is identified, where there are no sign constraints placed on the Preisach densities. The level of discretization M for the Preisach plane is also chosen to be 20, but now is for the entire input range $[T'_{\min}, T'_{\max}] = [21, 84]^\circ\text{C}$. Fig. 5.6 shows the modeling error, which is larger than the proposed model. It is interesting to notice that the negative density values of the signed Preisach operator are primarily located at the $\beta = \alpha$ line (see Fig. 5.7). This provides support for the proposed model; after all, the non-hysteretic, negative component in the proposed model that accounts for differential thermal expansion could be represented as negative densities located on the $\alpha = \beta$ line.

For the single-valued nonlinear approximation, a polynomial of degree 12 is chosen, the coefficients of which are found through a polynomial fitting between curvature measurements and the model predictions. It is not surprising that this model results in the largest modeling error among the three models explored, which is around 500 m^{-1} (Fig. 5.8). This model fails to account for the hysteresis effect.

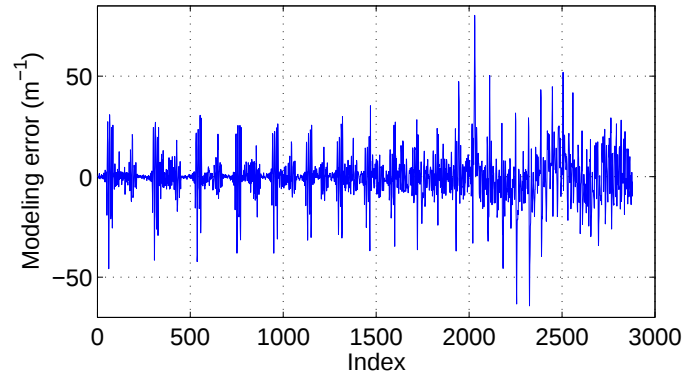


Figure 5.6: Modeling error with the signed Preisach operator for the entire temperature sequence.

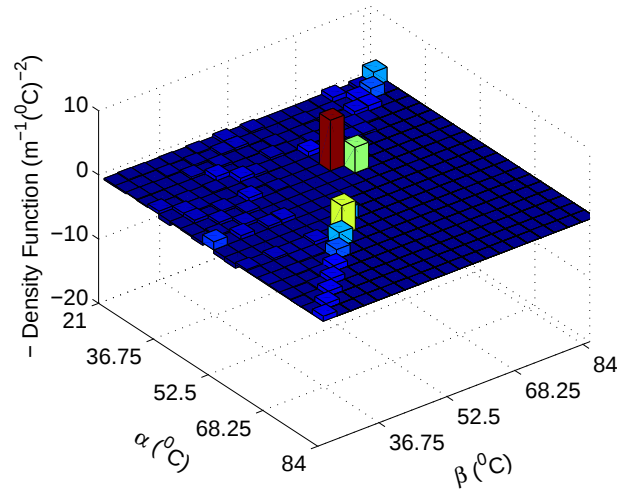


Figure 5.7: The negative of the identified density values for the signed Preisach operator. The negative is taken here so that the negative elements of the density function can be seen (on top); the positive elements are now flipped to the bottom of the plane, which are not visible here.

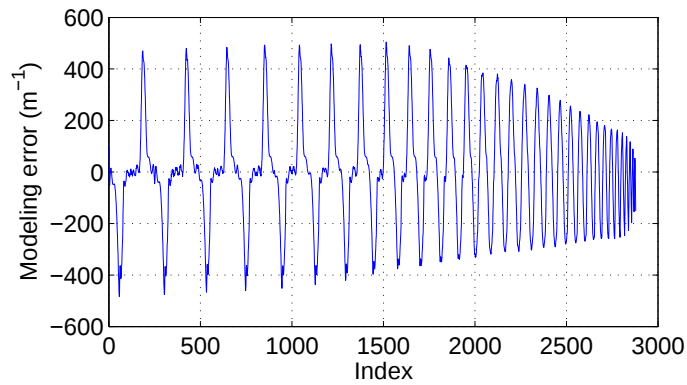


Figure 5.8: Modeling error with a polynomial model for the entire temperature sequence.

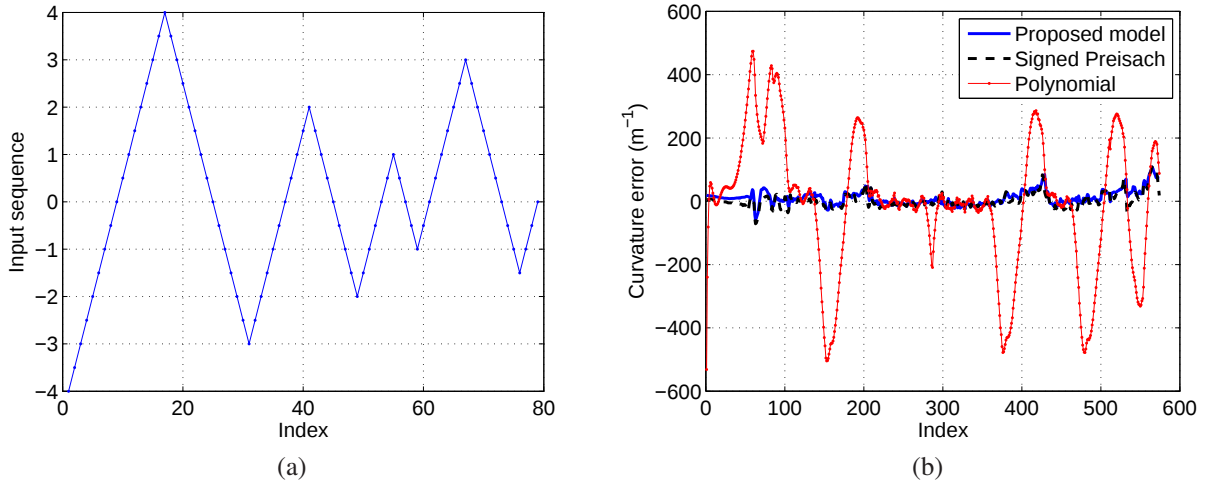


Figure 5.9: (a): A randomly chosen temperature input sequence for model validation. (b): Errors in predictions by different models under the input sequence.

In order to further validate the proposed model, the VO_2 actuator is subjected to a randomly chosen temperature sequence, shown in Fig. 5.9(a), and the output curvature data (not shown) is then collected. Fig. 5.9(b) shows the model prediction errors. The maximum prediction errors are 108.1, 108.8, and 478.1 m^{-1} for the proposed model, the signed Preisach operator, and the polynomial model, respectively. It is clear that, with an error of less than 5.6% over the curvature range of 1950 m^{-1} , the proposed model is effective in capturing the complex, hysteretic behavior of the VO_2 actuator. While the signed Preisach operator is able to provide a modeling accuracy comparable with the proposed model, it is not amenable to efficient inversion and will not be pursued further.

5.4 Inverse Compensation

There have been a number of inversion-related approaches for hysteresis compensation reported in the literature. For example, the direct inverse Preisach operator method [88] aims to identify a Preisach operator as the inverse operator based on empirical output-input data. The empirical

temperature vs. curvature hysteresis loops for VO₂ actuators (with the horizontal/vertical axes in Fig. 5.4(a)) exhibit sharp slope changes, making it difficult to approximate by a Preisach operator even with a prohibitively fine discretization scheme. Iterative Learning Control [89, 90] has proven effective in compensating both hysteresis and dynamics with relatively low requirement on modeling accuracy, but it only applies to periodic references and requires sensory feedback for the learning process. While the proposed inversion algorithm is noncausal, only the next desired output and the input history are required to obtain the input.

We propose to compensate the hysteresis effect in VO₂ by constructing a properly defined inverse of the model presented in earlier sections. The non-monotonic nature of the proposed hysteresis model presents several challenges in the inversion problem. In the continuous-time setting, there is great difficulty in establishing the existence and/or the uniqueness of a continuous input function given a desired, continuous output function. In the discrete-time setting, which is the case of practical interest, the concept of time-continuity is no longer relevant since the desired output function is given as a sequence of values for the operator to achieve, and input interpolation between the sampling times is typically used to realize a quasi-continuous output for the physical system. For a monotonic hysteresis operator, for any desired output value (within the output range) $y_d[n+1]$ at next time instant $n+1$, there always exists an input value $v[n+1]$ for $n+1$, such that, if the input varies monotonically from the current value $v[n]$ to $v[n+1]$, the output y would also change monotonically from its current value $y[n]$ to $y[n+1]$, which is equal or close to $y_d[n+1]$. This, however, is no longer true for a non-monotonic hysteresis operator as the one considered in this work – One may not find a single input value, monotonic interpolation to which would result in the desired output value. Therefore, a *sequence* of input values needs to be found to achieve a given desired output value; without proper constraints, the latter problem would admit infinitely many solutions. A constraint on this problem is imposed to minimize the implementation complexity

and assure proper output behavior: the number of elements in the computed input sequence is minimal. Specifically, for the proposed non-monotonic hysteresis model, the inversion problem is formulated as follows: given the current initial memory curve $\psi^{(0)}$ for the operator Ω , with the associated temperature input $T^{(0)}$, operator output $\kappa^{(0)} = \Omega[T^{(0)}; \psi^{(0)}]$, and a target output value $\bar{\kappa}$, find a new input sequence $\bar{T}$ with minimal number of elements, such that the final value of $\Omega[\bar{T}; \psi^{(0)}]$ is equal to $\bar{\kappa}$.

The proposed inversion algorithm is adapted from the one in [9] for the inversion of a monotonic Preisach operator with a piecewise constant density function. The algorithm in [9] exploits the monotonicity of the operator and the piecewise constant nature of the density, and finds the desired input by monotonically varying the input iteratively. Assuming that the input is being increased, with the value of the k -th iteration being $T^{(k)}$ and the corresponding memory curve being $\psi^{(k)}$, another increment of d ($d \leq \min\{d_1^{(k)}, d_2^{(k)}\}$) in the input would result in the following change in the output of the Preisach operator:

$$\Gamma[T^{(k)} + d; \psi^{(k)}] - \Gamma[T^{(k)}; \psi^{(k)}] = a_2^{(k)} d^2 + a_1^{(k)} d, \quad (5.13)$$

where $d_1^{(k)} > 0$ is such that $T^{(k)} + d_1^{(k)}$ would equal the next discrete input level, and $d_2^{(k)} > 0$ is such that $T^{(k)} + d_2^{(k)}$ would erase the next corner of the memory curve; see Fig. 5.10 for illustration. In (5.13), $a_1^{(k)}$ and $a_2^{(k)}$ are nonnegative constants associated with $\psi^{(k)}$ and the density values.

The core idea in [9] for inverting the Preisach operator is that, if $d \leq \min\{d_1^{(k)}, d_2^{(k)}\}$ can be found that solves

$$a_2^{(k)} d^2 + a_1^{(k)} d = \bar{y} - \Gamma[T^{(k)}; \psi^{(k)}], \quad (5.14)$$

where $\bar{y}$ is the desired output, then the required input is obtained as $T^{(k)} + d$; otherwise, let

$T^{(k+1)} = T^{(k)} + \min\{d_1^{(k)}, d_2^{(k)}\}$ and continue the iteration.

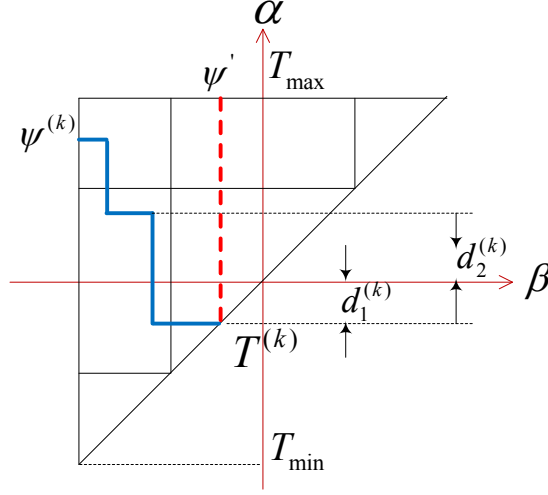


Figure 5.10: Illustration of the variables $d_1^{(k)}$ and $d_2^{(k)}$ used in inversion.

Since the curvature output κ starts to decrease (increase, resp.) when $T > T_{\max}$ ($T < T_{\min}$, resp.) because κp becomes positively (negatively, resp.) saturated, the maximum range of curvature output can be achieved by restricting the temperature T to the range $[T_{\min}, T_{\max}]$.

The curvature contribution from differential thermal expansion is given by:

$$\begin{aligned} \kappa_E &= Q[T] \triangleq \left(k_0 - \frac{(k_1 - k_0)T_{\min}}{T_{\max} - T_{\min}} \right) T_0 \\ &\quad - T \left(k_0 - \frac{T_0 + T_{\min}}{T_{\max} - T_{\min}} (k_1 - k_0) \right) \\ &\quad - T^2 \frac{k_1 - k_0}{T_{\max} - T_{\min}}. \end{aligned} \quad (5.15)$$

Consequently, the change in κ_E for an increment d at $T^{(k)}$ is obtained as:

$$Q[T^{(k)} + d] - Q[T^{(k)}] = -c_2 d^2 - c_1^{(k)} d, \quad (5.16)$$

where $c_1^{(k)} = k_0 - \frac{T_0 + T_{\min}}{T_{\max} - T_{\min}}(k_1 - k_0) + \frac{2(k_1 - k_0)}{T_{\max} - T_{\min}}T^{(k)}$, and $c_2 = \frac{k_1 - k_0}{T_{\max} - T_{\min}}$. Based on $T_0 \leq T_{\min} \leq T$ and $k_1 > k_0$ (Chapter 5.2.2), it can be shown that both $c_1^{(k)}$ and c_2 are positive.

Combining (5.13) and (5.16), we obtain:

$$\Omega[T^{(k)} + d; \psi^{(k)}] - \Omega[T^{(k)}; \psi^{(k)}] = (a_2^{(k)} - c_2)d^2 + (a_1^{(k)} - c_1^{(k)})d. \quad (5.17)$$

While $a_1^{(k)} \geq 0$, $a_2^{(k)} \geq 0$, $c_1^{(k)} > 0$, $c_2 > 0$, the signs of both $a_1^{(k)} - c_1^{(k)}$ and $a_2^{(k)} - c_2$ could be either positive or negative, which implies that an increment d in the temperature does not necessarily lead to an increase in curvature. Analogous statements can be made when the input decreases. Take a special example of the input being increased to some value T' (with the corresponding memory curve ψ') and then decreased by a small $d > 0$, in which case $a_1^{(k)} = 0$, and $\Omega[T' - d; \psi'] - \Omega[T'; \psi'] = (a_2^{(k)} - c_2)d^2 + c_1d$. Since the linear term dominates the quadratic term for small d , the output κ will increase immediately following the reversing of T at T' . Similarly, the curvature will decrease immediately following the reversing of a decreasing input. Interestingly, these predictions are confirmed by the experimental data, as can be seen clearly from Fig. 5.4(a). The following proposition will be instrumental in developing the inversion algorithm.

Proposition 4. *Consider the non-monotonic hysteresis model Eq. (5.8). Let $\kappa_{\max}$ and $\kappa_{\min}$ denote the maximum output and minimum output of the model, respectively. Then $\kappa_{\max}$ can always be achieved by first increasing the temperature T to $T_{\max}$ and then decreasing it monotonically to some value, and $\kappa_{\min}$ can always be achieved by decreasing T to $T_{\min}$ and then increasing it to some value.*

Proof. For any temperature T , the memory curve consisting of a single vertical segment (intersecting the line $\alpha = \beta$ at T) dominates any other memory curves at the same temperature, in the sense that the corresponding set $P_+ \triangleq \{(\beta, \alpha) : \gamma_{\beta, \alpha} = +1\}$ in the Preisach plane is maximal for

the given T . Consequently, the corresponding output κ_P of the Preisach operator (with nonnegative density function) is largest under the dominant memory curve. See Fig. 5.10 for illustration; and compare the two memory curves ψ' (dominant) and $\psi^{(k)}$ for the same temperature $T^{(k)}$.

The contribution κ_E depends only on the current temperature (no memory). Therefore, for a given temperature T , the maximum curvature output κ can always be achieved with an input sequence that results in a dominant memory curve at T . Such a dominant memory curve is created by decreasing the temperature from $T_{\max}$ to T . The other case follows from a similar argument. ■

The curvature output κ at $T_{\max}$ and $T_{\min}$ is denoted as κ_+ and κ_- , respectively. Due to the positive/negative saturation, κ_+ and κ_- are uniquely defined and independent of the initial condition of the operator Ω . The following assumption is made:

$$\text{Assumption 1 : } \kappa_{\min} < \kappa_- \leq \kappa_+ \leq \kappa_{\max}. \quad (5.18)$$

The assumption is expected to hold for all VO_2 -based micro bending actuators. In particular, for the sample used in this work $\kappa_{\max} = 1836 \text{ m}^{-1}$, $\kappa_+ = 1828 \text{ m}^{-1}$, $\kappa_- = -20 \text{ m}^{-1}$, and $\kappa_{\min} = -75 \text{ m}^{-1}$.

5.4.1 Inverse Compensation Algorithm

For brevity purposes, only the case $\bar{\kappa} > \kappa^{(0)}$ is discussed; the case $\bar{\kappa} < \kappa^{(0)}$ is treated in a symmetric manner. If $\bar{\kappa} > \kappa^{(0)}$, the discussion is divided into two sub-cases: (1) $\bar{\kappa} \leq \kappa_+$, and (2) $\kappa_+ < \bar{\kappa} \leq \kappa_{\max}$. For the first case, an iterative procedure modified from [9] is detailed; for the second case, the required input will be a two-step sequence and the procedure is outlined. Specifically,

If $\bar{\kappa} \leq \kappa_+$:

- Step 1: $k := 0$;

- Step 2:

1. Determine $d_1^{(k)} > 0$ such that $T^{(k)} + d_1^{(k)}$ equals the next discrete input level;
2. Determine the minimum $d_2^{(k)} > 0$ such that $T^{(k)} + d_2^{(k)}$ would erase the next corner of the memory curve $\psi^{(k)}$, which is generated under the iterative input sequence $\{T^{(0)}, T^{(1)}, \dots, T^{(k)}\}$;
3. Evaluate the coefficients $a_1^{(k)}, a_2^{(k)}, c_1^{(k)}, c_2$ for Eq. (5.13) and Eq. (5.16), whose values are determined by $\psi^{(k)}$ and $T^{(k)}$ and vary from iteration to iteration. Solve the equation

$$\begin{aligned} & \bar{\kappa} - \Omega[T^{(k)}, \psi^{(k)}] \\ &= (a_2^{(k)} - c_2)d^2 + (a_1^{(k)} - c_1^{(k)})d \end{aligned} \quad (5.19)$$

for d . If Eq. (5.18) has two positive solutions, let $d_0(k)$ be the smaller solution; if Eq. (5.18) has one positive solution, let $d_0^{(k)}$ equal that solution; if Eq. (5.18) has no positive solutions, let $d_0^{(k)} = 1000$ (a number larger than $(T_{\max} - T_{\min})/M$ but otherwise arbitrary); This scheme ensures the uniqueness of the solution.

4. Let $d^{(k)} =: \min\{d_0^{(k)}, d_1^{(k)}, d_2^{(k)}\}$, $T^{(k+1)} = T^{(k)} + d^{(k)}$, $\kappa^{(k+1)} = \Omega[T^{(k+1)}; \psi^{(k)}]$;
5. If $d^{(k)} = d_0^{(k)}$, go to Step 3; otherwise let $k := k + 1$ and go back to Step 2.

- Step 3: $\bar{T} := T^{(k+1)}$ and stop.

If $\kappa_+ < \bar{\kappa} \leq \kappa_{\max}$: Inversion can be realized by first applying the temperature $T_{\max}$ to saturate the Preisach operator, and then decreasing input iteratively following a similar scheme.

Note that the inversion algorithm requires knowing the current initial memory curve $\psi^{(0)}$. As for the inversion of a Preisach operator [27], this requirement is typically satisfied by setting the initial memory curve at time $n = 0$ to a (known) configuration that corresponds to positive or negative saturation of the Preisach operator Γ , by applying $T_{\max}$ or $T_{\min}$, respectively. The memory curve at any future time $n > 0$ can then be inferred based on the curve at $n - 1$ and the sequence of input values applied after time $n - 1$. The following proposition summarizes the properties of the proposed inversion algorithm.

Proposition 5. *The proposed inversion algorithm produces the input $\bar{T}$ satisfying $\Omega[\bar{T}; \psi^{(0)}] = \bar{\kappa}$ in no more than $n_c(\psi^{(0)}) + M$ iterations, where $n_c(\psi^{(0)})$ denotes the number of corners in $\psi^{(0)}$, M is the discretization level. Furthermore, $\bar{T}$ has no more than two elements.*

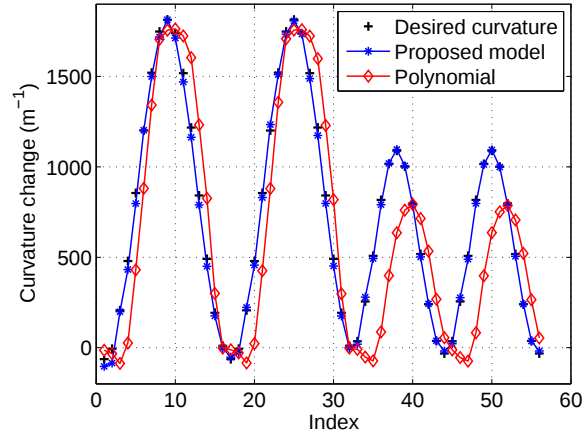
Proof. The case where $\bar{\kappa} > \kappa^{(0)}$ is proved in detail. First, consider the case $\bar{\kappa} < \kappa_+$. Since the operator Ω is continuous (i.e., its output changes continuously with the input), there must exist $\bar{T} \in [T^{(0)}, T_{\max}]$ such that $\Omega[\bar{T}; \psi^{(0)}] = \bar{\kappa}$. The inversion algorithm then searches for the exact solution in contiguous segments within $[T^{(0)}, T_{\max}]$, where the segments are defined by the discrete input levels and the memory curve $\psi^{(0)}$. The number of such segments is no greater than $n_c(\psi^{(0)}) + M$, which provides the upper bound for the iteration steps. The case of $\kappa_+ < \bar{\kappa} \leq \kappa_{\max}$ can be proved following similar and simpler arguments. Applying first $T_{\max}$ erases all the memory curve corners, so it is only needed to search within segments defined by the discrete input levels. The maximum iteration steps will be M in this case. The last statement of the proposition is evident from the description of the algorithm. ■

From Proposition 5, the efficiency of the proposed inverse compensation algorithm in this work is comparable to that of the inversion algorithm for a Preisach operator [9].

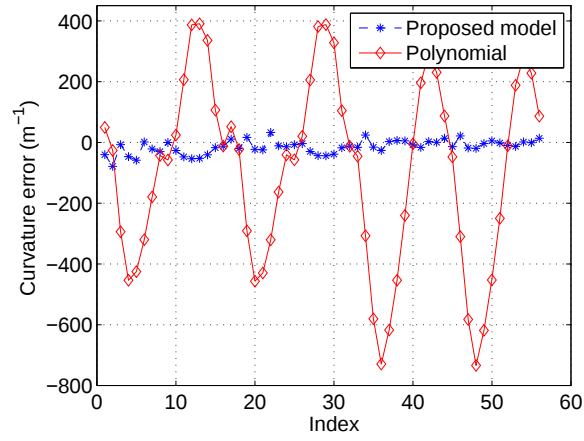
5.4.2 Experimental Validation

The performance of the proposed inversion algorithm has been examined in open-loop curvature tracking experiments. For comparison, the single-valued polynomial model has also been inverted through a look-up table. The desired curvature is chosen to be from -63 m^{-1} to 1814 m^{-1} to test the effectiveness of the inverse compensation algorithm for a wide curvature range.

Fig. 5.11(a) shows the curvature outputs obtained under the two inversion schemes. Fig. 5.11(b) shows the corresponding inversion errors. The inversion of the proposed model is proven to be effective, with the largest curvature error of 78 m^{-1} , which is only 4.1% of the whole curvature range. In comparison, the inversion of the non-hysteretic polynomial model produces a maximum error of 734.2 m^{-1} . The RMSE value is also calculated to quantify the tracking error. The RMSE of the proposed inversion is only 26.7 m^{-1} , compared to 320.6 m^{-1} for the polynomial case. The last observation from Fig. 5.11(b) is that when the magnitude of the desired curvature changes, the proposed inversion scheme can still maintain a small curvature error, while the inversion based on the polynomial model produces a large error.



(a)



(b)

Figure 5.11: (a): Open-loop inverse control performance for the proposed model and the polynomial model. (b): Inverse compensation errors.

Chapter 6

Modeling and Inverse Compensation of Hysteresis using an Extended GPI (EGPI) model

In this chapter, an EGPI model is proposed to capture sophisticated hysteresis as observed in VO_2 . The model consists of a nonlinear memoryless function and a GPI model, the play operators of which have the same envelope functions. The EGPI model is tested in modeling asymmetric and non-monotonic hysteresis between the curvature output and the temperature input of a VO_2 -coated microactuator, demonstrating 40% less modeling error than a GPI model. The advantages of the proposed model are further verified in modeling the asymmetric, partially saturated hysteresis between the resistance output and the temperature input of a VO_2 film. A novel inversion algorithm is then derived based on the fixed-point iteration framework. The convergence condition of the proposed algorithm is further derived. Finally, both simulation and experimental results are provided to support the effectiveness of the inversion algorithm.

6.1 EGPI model for Non-monotonic Hysteresis

The GPI model can capture asymmetric hysteresis with saturation, and it has an analytical inversion [28] as long as the envelope functions of all the generalized play operators are of the same form.

We propose adding a nonlinear memoryless function $D(\cdot)$ to the GPI model:

$$u(t) = D(v(t)) + \sum_{j=0}^N p(r_j) F_{r_j}^\gamma[v](t), \quad (6.1)$$

This model is called the EGPI model. Fig. 6.1 shows a GPI model and an EGPI model with identical weights of generalized play operators. It is shown that the EGPI model can better model complex hysteresis. By choosing appropriate memoryless functions, the EGPI model can also capture non-monotonic hysteresis.

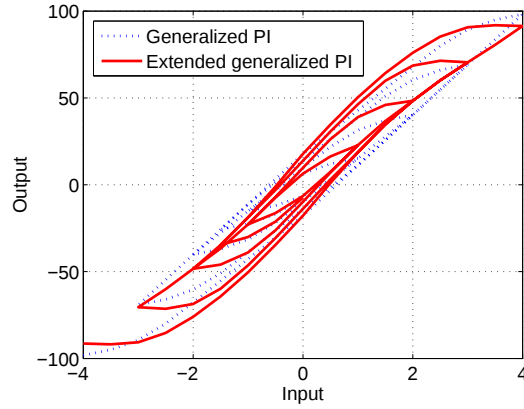


Figure 6.1: A GPI model and an EGPI model with identical weights of generalized play operators.

6.2 Inverse Compensation Algorithm

The goal of inverse compensation is to cancel out the hysteresis nonlinearity by constructing an inverse hysteresis model. A fixed-point iteration-based inversion algorithm for an EGPI model is proposed.

Denote u_d as the desired output of the EGPI model. Then the EGPI model is expressed as

$$u_d = D(v) + \Psi[v]. \quad (6.2)$$

where $D(v)$ is the memoryless component and $\Psi[v]$ is the GPI hysteresis model. Since the inversion of $\Psi[v]$ is available [28], rewrite Eq. (6.2) as

$$v = \Psi^{-1}[u_d - D(v)]. \quad (6.3)$$

Unlike the inversion of GPI model, $u_d - D(v)$ is used as the input for the inverse EGPI model. The right part of the above equation can be solved with a known input v ; however, v is also the desired solution. To solve the problem, we first recall some background materials.

Contraction mapping [91]: Let $(X, \mathbf{d})$ be a metric space. A map $T : X \rightarrow X$ is called a contraction mapping on X , if there exists $q \in [0, 1)$ such that: $\mathbf{d}(\mathbf{T}(\mathbf{x}), \mathbf{T}(\mathbf{y})) \leq q\mathbf{d}(\mathbf{x}, \mathbf{y})$ for all $x, y \in X$.

Proposition 6. *Let $(X, \mathbf{d})$ be a non-empty complete metric space with a contraction mapping $T : X \rightarrow X$. Then T admits a unique fixed point x^* in X (i.e. $T(x^*) = x^*$). Furthermore, x^* can be found as follows: start with an arbitrary element x_0 in X and define a sequence x_n by $x_n = T(x_{n-1})$, then $x_n \rightarrow x^*$ [91].*

From Proposition 6, if $\Psi^{-1}[u_d - D(v)]$ is a contraction mapping in terms of v , the inversion can be obtained by iterating $v_k = \Psi^{-1}[u_d - D(v_{k-1})]$, $k = 1, 2, \dots, n, \dots$ until $|v_n - v_{n-1}| < \sigma$, $\sigma > 0$. The following proposition provides a sufficient condition for the convergence of the inversion algorithm.

Proposition 7. *Denote $\Psi^{-1}[u_d]$ as the inversion of the GPI model, where u_d is the desired output. Then the operator $\Psi^{-1}[u_d - D(v)]$ is a contraction mapping on $[v_{\min}, v_{\max}]$, if*

$$\min_v \left\{ \frac{d\gamma_R}{dv}, \frac{d\gamma_L}{dv} \right\} \cdot p(r_0) > \frac{dD}{dv}. \quad (6.4)$$

Proof. When $u_1 > u_2$:

$$\begin{aligned}
& |\Psi^{-1}[u_1] - \Psi^{-1}[u_2]| \\
&= |\gamma_R^{-1} \circ \Pi^{-1}[u_1] - \gamma_R^{-1} \circ \Pi^{-1}[u_2]| \\
&\leq \max_u \left\{ \frac{d\gamma_R^{-1}}{du} \right\} \cdot |\Pi^{-1}[u_1] - \Pi^{-1}[u_2]| \\
&\leq \max_u \left\{ \frac{d\gamma_R^{-1}}{du} \right\} \cdot \hat{p}(r_0) \cdot |u_1 - u_2| \\
&= \max_u \left\{ \frac{d\gamma_R^{-1}}{du} \right\} \cdot |u_1 - u_2| / p(r_0). \tag{6.5}
\end{aligned}$$

The second inequality of Eq. (6.5) holds, since

$$\begin{aligned}
& |\Pi^{-1}[u_1] - \Pi^{-1}[u_2]| \\
&= \Pi^{-1}[u_1] - \Pi^{-1}[u_2] \\
&= \hat{p}(r_0) \cdot (u_1 - u_2) + \sum_{i=1}^N \hat{p}(r_i) (F_{\hat{r}_i}[u_1](t) - F_{\hat{r}_i}[u_2](t)) \\
&\leq \hat{p}(r_0)(u_1 - u_2) \\
&= \hat{p}(r_0) \cdot |u_1 - u_2|. \tag{6.6}
\end{aligned}$$

The inequality of Eq. (6.6) holds since $\hat{p}(r_i) < 0$, for $i \geq 1$ (See Eq. (B.17) in Appendix) and

$$\Pi^{-1}[u_1] \geq \Pi^{-1}[u_2].$$

Similarly, for the case of $u_1 < u_2$:

$$|\Psi^{-1}[u_1] - \Psi^{-1}[u_2]| \leq \max_u \left\{ \frac{d\gamma_L^{-1}}{du} \right\} \cdot |u_1 - u_2| / p(r_0). \tag{6.7}$$

Combing the above two cases,

$$\begin{aligned}
& |\Psi^{-1}[u_d - D(v_1)] - \Psi^{-1}[u_d - D(v_2)]| \\
& \leq \max_u \left\{ \frac{d\gamma_R^{-1}}{du}, \frac{d\gamma_L^{-1}}{du} \right\} \cdot \frac{dD}{dv} \cdot |v_1 - v_2| / p(r_0) \\
& = \max_v \left\{ 1 / \frac{d\gamma_R}{dv}, 1 / \frac{d\gamma_L}{dv} \right\} \cdot \frac{dD}{dv} \cdot |v_1 - v_2| / p(r_0) \\
& = \frac{dD}{dv} \cdot |v_1 - v_2| / p(r_0) / \min_v \left\{ \frac{d\gamma_R}{dv}, \frac{d\gamma_L}{dv} \right\}. \tag{6.8}
\end{aligned}$$

Thus, $\Psi^{-1}[u_d - D(v)]$ is a contraction mapping when the following inequality is satisfied:

$$\min_v \left\{ \frac{d\gamma_R}{dv}, \frac{d\gamma_L}{dv} \right\} \cdot p(r_0) > \frac{dD}{dv}. \tag{6.9}$$

■

For CPI model, since $d\gamma_R/dv = d\gamma_L/dv = 1$, so the convergence condition degenerates to

$$p(r_0) > \frac{dD}{dv}. \tag{6.10}$$

For either CPI model or GPI model, since $dD/dv = 0$, no iteration is needed.

Note that, from Proposition 7, the convergence of the proposed algorithm depends on the parameters of the hysteresis model.

6.3 Experimental Results: Modeling

The modeling performances of the EGPI models involving the curvature and temperature hysteresis relationship of a VO₂ coated cantilever, and the resistance and temperature hysteresis relationship of a VO₂ film are shown.

6.3.1 Curvature-temperature Hysteresis of a VO₂-coated Microactuator

As discussed in Chapter 5, the VO₂-coated silicon micro-cantilevers are subject to two actuation effects when its temperature is varied. First, the stress due to thermally induced phase transition of VO₂ makes the beam bend towards the VO₂ layer, a process that is inherently hysteretic. Second, the differential thermal expansion effect generates stress in the opposite direction. As a result, the hysteresis between the bending curvature and the temperature is non-monotonic.

Following similar treatment as that in Chapter 5, a 172 nm thick VO₂ layer was deposited on a silicon cantilever with length of 300μm. The microcantilever was glued to a glass substrate that was directly in contact with a Peltier heater. A PSD and a laser were used to measure the deflection of the microcantilever. The curvature was then obtained based on the PSD measurement.

In order to capture the hysteresis, the envelope functions for the extended generalized play operator are chosen to be hyperbolic-tangent functions in the form of

$$\gamma_R(v(t)) = \tanh(a_R v(t) + b_R), \quad (6.11)$$

$$\gamma_L(v(t)) = \tanh(a_L v(t) + b_L). \quad (6.12)$$

The non-hysteretic component is expressed as

$$D(v(t)) = p_0 \sin(\omega \cdot v(t)) + c_1. \quad (6.13)$$

The number of the generalized play operators is chosen to be $N = 15$, and the play radii are chosen as $r = i/N, i = 0, 1, \dots, N - 1$. The parameters identified for the GPI model and the EGPI model are shown in Table 6.1. The weights of the GPI model and the EGPI model are different due to the effect of the nonlinear memoryless function. The identified weights $p(r_i), i = 0, 1, \dots, N - 1$

Table 6.1: Parameters of the GPI model and the EGPI model for hysteresis of a VO₂-coated microactuator.

	a_L	b_L	a_R	b_R	p_0	ω	c_1
Generalized	0.11	-5.19	0.13	-6.94	0	0	914.20
Extended	0.18	-9.61	0.14	-7.19	40.1	0.17	923.91

of both models are not included in the interest of brevity.

Note that given the envelope functions form, the identified EGPI model may not optimally model the hysteresis. The non-hysteretic component is identified as follows: first a GPI model is adopted to model the hysteresis, then the non-hysteretic component is chosen based on the remaining modeling error of the GPI model. The GPI model is modeled by the summation of weighted generalized play operators and an offset c_1 .

In order to cover the whole phase transition range, the temperature range was chosen to be from 20 °C to 80 °C. In particular, we varied the temperature in repeated heating-cooling cycles with the temperature range decreased for each cycle. Fig. 6.2(a) and (b) show the modeling performance of GPI model and that of the EGPI model, respectively. Compared with the GPI model, the proposed model can capture the asymmetric and non-monotonic hysteresis more accurately. The RMSE and the absolute maximum of the error are selected to quantify the modeling performance. The RMSE of the GPI model is 38.5 m⁻¹, and the RMSE of the EGPI model is 26.4 m⁻¹. The largest error of the GPI model is 148.9 m⁻¹, while that of the EGPI model is 89.6 m⁻¹. Therefore, the EGPI model can capture the asymmetric and non-monotonic hysteresis behavior more accurately, with 31 % and 40 % smaller error in terms of RMSE and the largest modeling error, respectively.

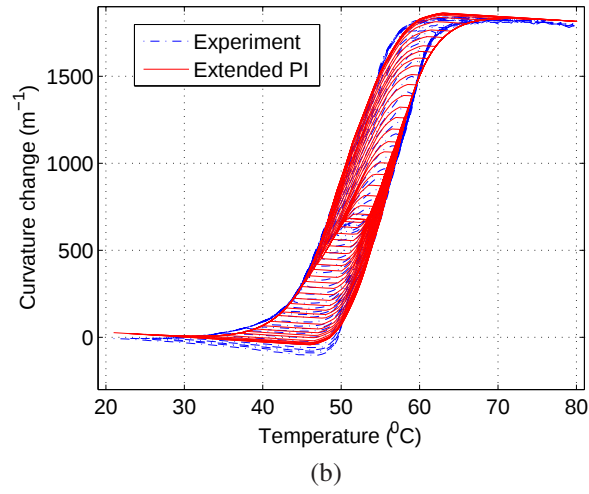
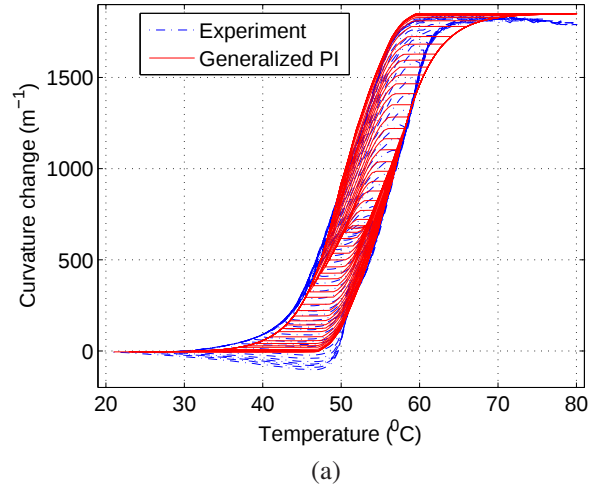


Figure 6.2: The performance of modeling curvature-temperature hysteresis of a VO_2 -coated microcantilever based on: (a) GPI model. (b) EGPI model.

Table 6.2: Parameters of the GPI model and the EGPI model for hysteresis of a VO₂ film.

	a_L	b_L	a_R	b_R	p_0	a_D	b_D	c_2
Generalized	0.14	-8.5	0.16	-9.5	0	0	0	-3.23
Extended	0.16	-10.3	0.20	-11.6	0.4	0.03	-1.6	-3.26

6.3.2 Resistance-temperature Hysteresis of a VO₂ Film

A VO₂ layer was deposited by pulsed laser deposition. The film was heated with a Peltier heater. The experimental setup in this work was similar to the one used in Chapter 2. The resistance of the film was measured through two aluminium contacts patterned on the VO₂ film.

Similarly, the temperature profile in time followed a pattern of damped oscillations. It is shown in Fig. 6.3 that the measured resistance (Z) changes by approximately two orders of magnitude. Furthermore, in order to have non-negative weights for the hysteresis models, $-\log_{10} Z$ is taken as the output. The hysteresis behavior shown in Fig. 6.3 is asymmetric and also partially saturated. Following [13, 28], the envelope functions are selected to be hyperbolic-tangent functions. The memoryless function is selected as the sum of a hyperbolic-tangent function and an offset:

$$D(v(t)) = p_0 \cdot \tanh(a_D v(t) + b_D) + c_2. \quad (6.14)$$

The number of play operator is chosen to be $N = 30$, and the radii are chosen to be $r = i/N, i = 1, \dots, N$. The GPI model is modeled by the summation of the same number of weighted generalized play operators and an offset c_2 . The identified parameters of the GPI model and the EGPI model are shown in Table 6.2. The weights of the GPI model and the EGPI model are different due to the effect of the nonlinear memoryless function. The identified weights of both models are not included in the interest of brevity.

Fig. 6.3 (a) and (b) show the modeling performance based on the GPI model and the proposed

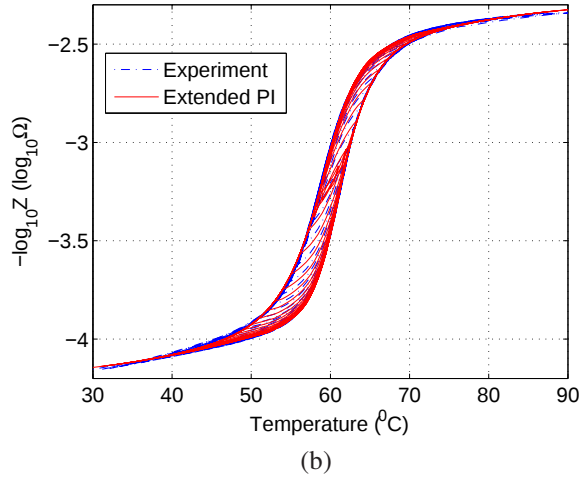
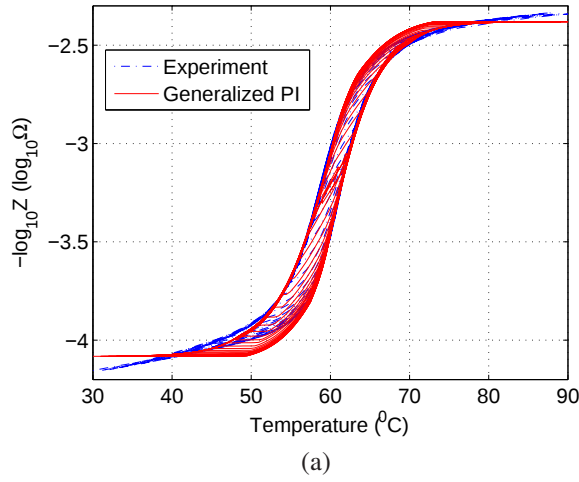


Figure 6.3: The performance of modeling the resistance-temperature hysteresis of a VO_2 film based on: (a) GPI model. (b) EGPI model.

model, respectively. The RMSE and the maximum absolute error of the GPI model are 0.031 and $0.082 \log_{10} \Omega$, respectively, while the corresponding values for the EGPI model are 0.012 and $0.041 \log_{10} \Omega$, respectively. The GPI model has 158% and 100% larger RMSE error and maximum absolute error, respectively.

Fig. 6.4 (a) and (b) show a random temperature sequence and its corresponding resistance output. The model estimation errors based on the GPI model and the EGPI model, respectively, are shown in Fig. 6.4 (c). The RMSE and the average absolute error of the GPI model are 0.034

and $0.027 \log_{10} \Omega$, respectively, while the corresponding values for the EGPI model are 0.012 and $0.009 \log_{10} \Omega$, respectively. The effectiveness of the EGPI model in capturing the asymmetric and partial saturated hysteresis is thus further demonstrated.

6.4 Inverse Compensation Results

Examples are shown to illustrate the effectiveness of the inverse algorithm both in simulation and experiments.

6.4.1 Simulation

Consider an EGPI operator expressed as a memoryless function and a CPI model, i.e.,

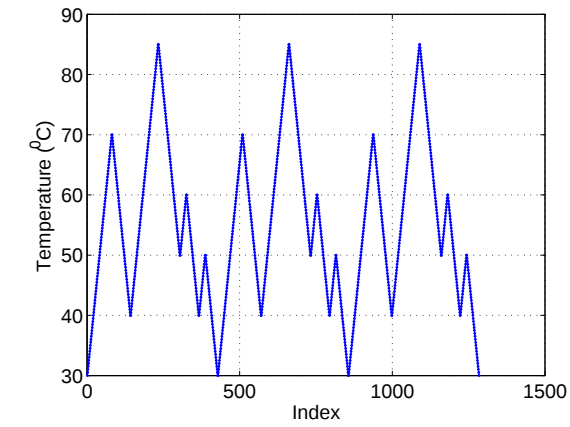
$$\gamma_R(v(t)) = v(t), \quad (6.15)$$

$$\gamma_L(v(t)) = v(t). \quad (6.16)$$

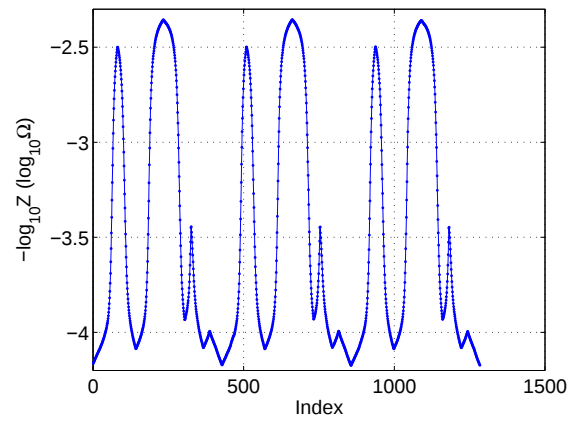
The memoryless component is chosen as

$$D(v(t)) = p_0 \cos(v(t)). \quad (6.17)$$

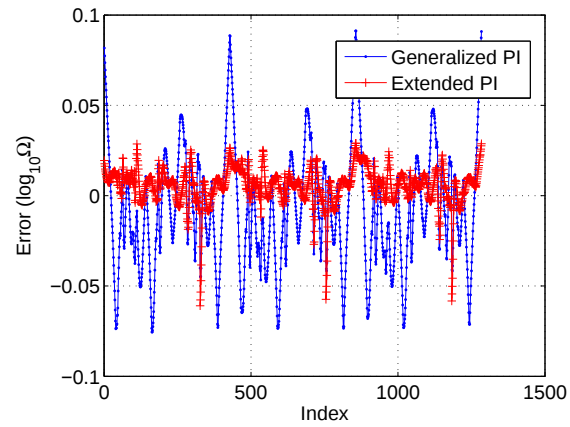
Note the envelope function and memoryless component are chosen in the above form as an illustrative example. The radii and their corresponding weights of the play operators are shown in Table III. $D(v(t))$ is chosen to be $5 \cos(v(t))$. The convergence condition for the given model is satisfied since



(a)



(b)



(c)

Figure 6.4: Model verification of the resistance-temperature hysteresis in a VO_2 film: (a) a random temperature sequence. (b) corresponding resistance output. (c) Modeling comparison between the GPI model and EGPI model.

Table 6.3: Parameter of the EGPI model

i	1	2	3	4	5	6
r_i	0	0.2	0.4	0.6	0.8	2
$p(r_i)$	6	2	1	2	4	4

$$\min\left\{\frac{d\gamma_R}{dv}, \frac{d\gamma_L}{dv}\right\} \cdot p(r_0) = 6 > 5 \geq \frac{dD}{dv}. \quad (6.18)$$

Fig. 6.5(a) shows a randomly chosen input sequence, and Fig. 6.5(b) shows the input and model output relationship. Fig. 6.6 (a) shows the inversion of the model based on the proposed algorithm, and Fig. 6.6(b) shows the resulting relationship between desired output and calculated output. The good linearity demonstrates the effectiveness of the inverse algorithm.

The average number of iterations is 8.38 when the convergence criterion σ is chosen to be 0.0001, which shows the efficiency of the algorithm. It is found in simulation that if σ is enlarged to be 0.01, the average number of iteration decreases to 5.31. It is also verified that when the value of p_0 is reduced, the inversion algorithm may not converge; on the other hand, if $p(r_0)$ remains sufficiently large, the EGPI algorithm will always converge.

6.4.2 Experimental Verification

The proposed inversion algorithm is also tested in experiments to compensate the resistance-temperature hysteresis in the VO₂ film. It is verified that when the number of generalized play operators is more than 5, the modeling performance will not improve significantly while incurring higher computational cost. Therefore, a simpler and more efficient model with 5 generalized play operators and a nonlinear term is utilized. The parameters can be found in Table 3.6 and [82].

It is found that when $v \in [43.2, 74.2]$ °C, the convergence requirement will be satisfied.

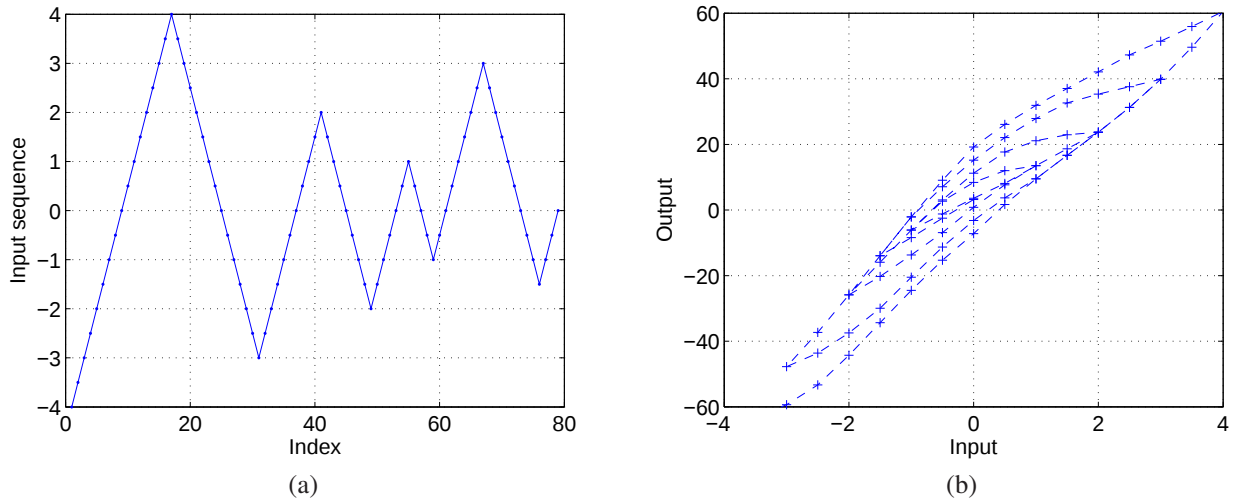


Figure 6.5: Simulation verification of the inverse algorithm. Hysteresis relationship: (a) Input sequence. (b) Input-output of the EGPI model.

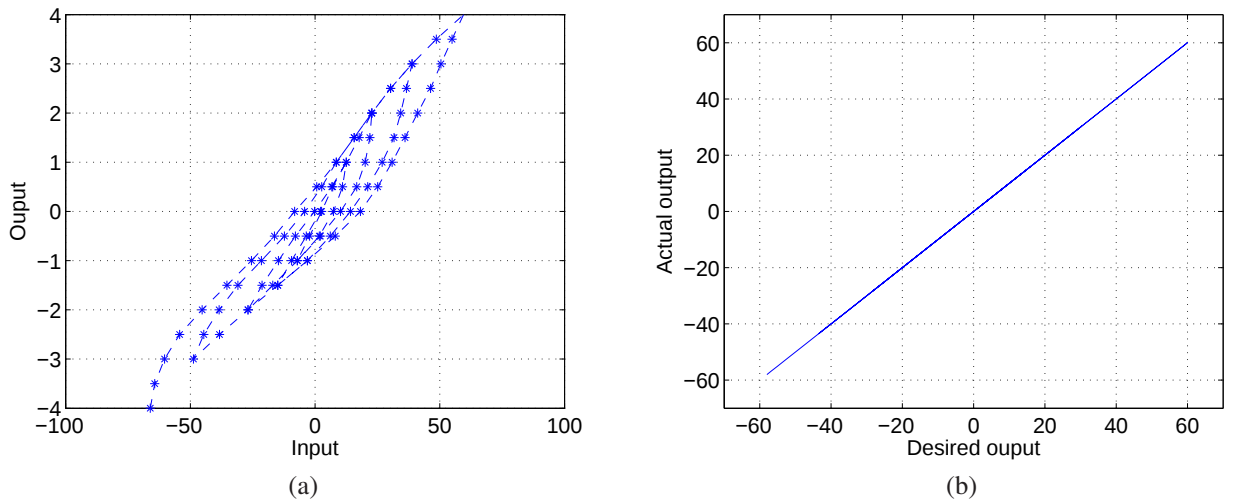
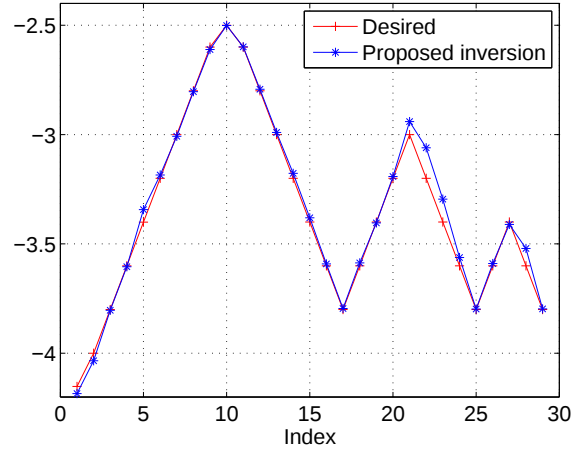
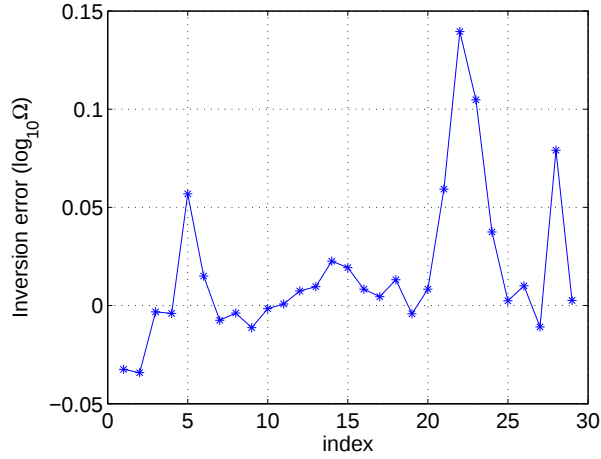


Figure 6.6: Compensation of hysteresis in simulation: (a) Input-output of the inverse EGPI model. (b) The relationship of the desired output and the actual output after hysteresis compensation.



(a)



(b)

Figure 6.7: (a). Inverse compensation performance in experiment. (b). Inversion compensation error for the EGPI model.

Note that this covers the typical operating range for VO_2 . Outside the region $[43.2, 74.2]^\circ\text{C}$, the convergence requirement may fail, since when $v \rightarrow \infty$, $d\gamma/dv \rightarrow 0$ faster than dD/dv , and $d\gamma/dv \ll dD/dv$, thus making Eq. (6.4) difficult to meet.

Fig. 6.7 shows the inverse compensation performance and the inversion error. The absolute maximum inversion error is around $0.135 \log_{10} \Omega$, which still demonstrates the effectiveness of the proposed compensation approach.

Chapter 7

A Composite Hysteresis Model in Self-Sensing Feedback Control of VO₂-integrated Microactuators

In this chapter a composite hysteresis model is proposed for self-sensing feedback control of VO₂-integrated microactuators. The deflection of the microactuator is estimated with the resistance measurement through the proposed model. To capture the complicated hysteresis between the resistance and the deflection, we exploit the physical understanding that both the resistance and the deflection are determined by hysteretic relationships with the temperature. Since direct temperature measurement is not available, the concept of temperature surrogate, representing the constant current value in Joule heating that would result in a given temperature at the steady state, is explored in the modeling. In particular, the hysteresis between the deflection and the temperature surrogate and the hysteresis between the resistance and the temperature surrogate are captured with a generalized Prandtl-Ishlinskii (GPI) model and an extended generalized Prandtl-Ishlinskii (EGPI) model, respectively. The composite self-sensing model is obtained by cascading the EGPI model with the inverse GPI model. For comparison purposes, two algorithms, based on a Preisach model and an EGPI model, respectively, are also used to estimate the deflection based on the resistance measurement directly. The proposed self-sensing schemes is evaluated with proportional-integral

(PI) control of the microactuator under step and sinusoidal references, and its superiority over the other schemes is demonstrated by experimental results.

7.1 Experimental Procedures

7.1.1 VO₂-integrated Actuator Fabrication

The microactuator used in this work consisted of a silicon dioxide (SiO₂) microcantilever with patterned VO₂ film inside the structure. The fabrication process flow for this device is shown in Fig. 7.1. The process starts with the deposition of 1 μm layer of SiO₂ using plasma-enhanced chemical vapor deposition (PECVD) at temperature of 300 °C on a 300 μm thick silicon (Si) wafer. This SiO₂ layer was used as the substrate to generate VO₂ with highly oriented crystalline structure, to achieve maximum actuation effect [92]. A VO₂ layer (270 nm) was deposited by pulsed laser deposition (PLD) [56] and patterned with reactive ion etch (RIE). The patterned VO₂ film was used as the active actuation element in the cantilever. Another SiO₂ layer (400 nm) was deposited by PECVD at 250 °C to isolate the VO₂ from the metal layer (to be processed next) and patterned with RIE to open the contact to the VO₂. The lower temperature was used to mitigate the adverse effects of exposing VO₂ films to high temperatures. Two openings on the top side of the SiO₂ were made to expose the VO₂ in selected regions. Two Ti (40 nm)/Au (160 nm) layers were deposited by evaporation and patterned by lift-off techniques. The first one was to partially fill the opening in the SiO₂, and the second one was to form the heating element and the traces for the VO₂ resistance contacts. Certain areas of SiO₂ with thickness of 1.4 μm were etched with RIE to define the geometry of the cantilever and expose the Si for the releasing step. XeF₂ gas was used to do an isotropic etch of the Si and release the cantilever.

In these devices, the VO₂ film was fully integrated in the fabrication process flow of the device.

The gold metal layer used for both resistance measurement and actuation of the VO₂ film was deposited at room temperature after the VO₂ film was deposited and patterned. This resulted in a direct electrical contact between the metal layer and a uniform VO₂ film. In previous Chapters, the VO₂ film was deposited at high temperatures on platinum contact pads that were accessed by vias through a SiO₂ film that separated the metal layer and the VO₂ film in regions other than the contact pad. This not only created a step in the VO₂ film thickness at the electrical contact, but also a VO_x (x different than 2) layer between the film and the contact pad. Thus, the VO₂ films used in this chapter were of better quality in terms of uniformity and stoichiometry, and the resistance measurements on the fully-integrated VO₂ devices included only the VO₂ thin film.

Note that the actuator is a bimorph bender consisting of VO₂ and SiO₂ layers. Thermally induced phase transition in VO₂ will generate internal stress that causes drastic bending of the structure toward the VO₂ layer. In addition, differential thermal expansion of the two materials results in an opposite bending effect. The combination of the two actuation effects leads to a non-monotonic hysteretic behavior between the deflection and the temperature.

7.1.2 Experimental Setup

The experimental setup was same as Fig. 4.7(a). The system is based on the laser scattering technique, using an IR laser ($\lambda=808$ nm) and a position sensitive detector (PSD) to track the displacement of the microactuator (shown in Fig. 7.2). A charge couple device (CCD) camera was used for alignment and calibration purposes. Note that while the CCD camera has a limited pixel resolution (1.3 μm), the resulting calibration error mainly introduces a scaling factor close to 1 for the relative displacement measurement, and thus it has minimal impact on the characterization and comparison of different self-sensing schemes in this work. A dSPACE system was used for data acquisition and control implementation. The power of the sensing laser (222 mW) was calibrated

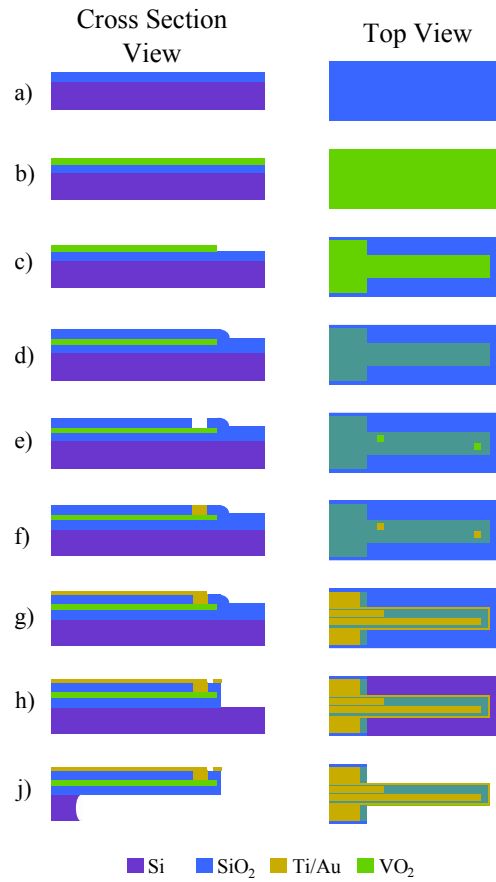


Figure 7.1: Fabrication process flow for the VO₂-integrated actuator. a) Deposition of SiO₂ (1 μ m) by PECVD; (b) deposition of VO₂ (270 nm) by PLD; (c) patterning (etch) of VO₂ by RIE; (d) deposition of SiO₂ (0.4 μ m) by PECVD; (e) patterning (etch) of SiO₂ by RIE; (f-g) deposition of Ti/Au by evaporation and patterning by lift-off; (h) RIE of SiO₂ for device pattern; (j) cantilever released by XeF₂ isotropic etching of Si.

to be the minimum possible to be sensed by the PSD without heating the cantilever due to photon absorption. The voltage output (V_D) of the PSD was linearly proportional to the position of the laser. With the images captured by the CCD camera, this voltage (V_D) was mapped to the deflection of the microactuator. The chip was inside a side braze packaging (wire-bonded), which was connected to the dSPACE. The current I_H shown in Fig. 4.7(b) was used to control the temperature of the microactuator by Joule heating. The current was generated using two resistances in series: the heater resistance and an external resistance, whose only purpose was to limit the maximum current that can be applied to the system. The VO_2 resistance (R_V) was measured *in situ* by using a constant current and monitoring the voltage across the resistance – the magnitude of the constant current ($21\ \mu\text{A}$) was chosen so that it would not heat the VO_2 considerably, but could be measured by the dSPACE system.

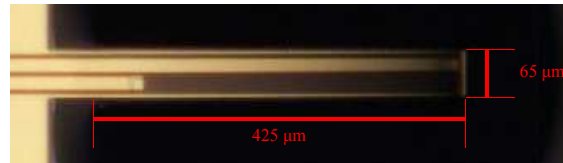


Figure 7.2: The VO_2 -integrated microactuator used in this work, with length $425\ \mu\text{m}$ and width $65\ \mu\text{m}$.

7.1.3 Measurement of Hysteretic Behavior

In order to obtain the hysteresis measurement, a sequence of quasi-static input values are applied, and for each input value, the corresponding output (resistance or deflection) at the steady state is recorded. In this work, the term “index” refers to the numbering of the quasi-static input values as well as that of the corresponding steady-state output values. Fig. 7.3(a) shows the current input with the form of damped oscillations. The measurement was taken under a quasi-static condition, where each current value was held for 10 ms since the heating dynamics had a time

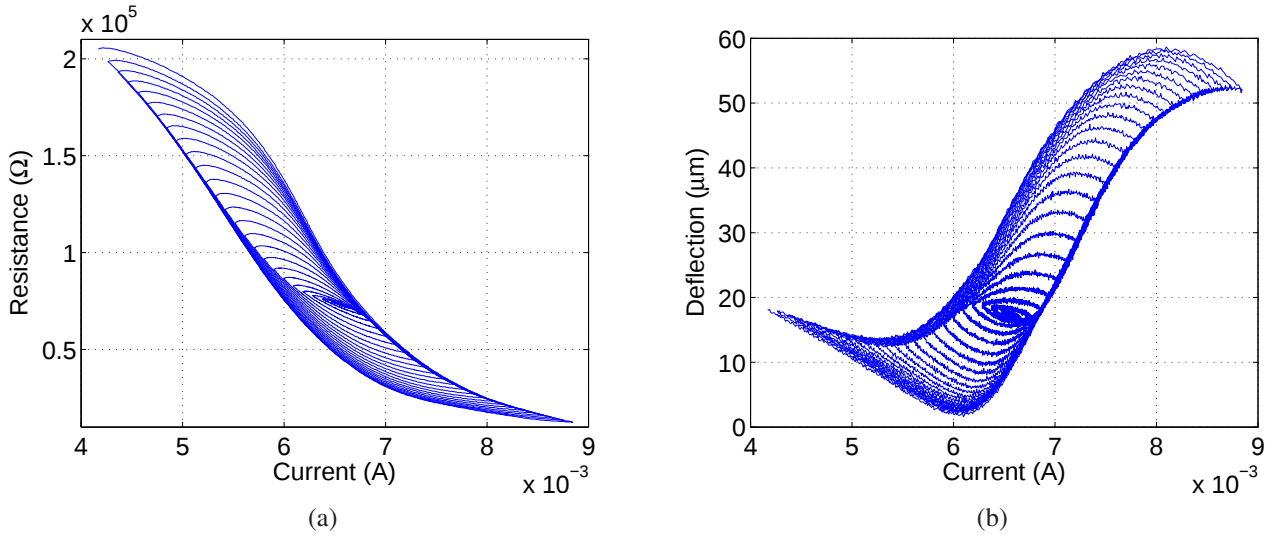


Figure 7.3: (a) The hysteresis between the resistance and the current; (b) the hysteresis between the deflection and the current.

constant of less than 2 ms (see Section V). Fig. 7.3(b) shows the corresponding resistance of the VO_2 microactuator. The total resistance range is $[11.19, 231.75]$ k Ω , with the current ranging from 3.68 mA to 8.49 mA. There is asymmetric hysteresis between the resistance and the current, which shows a monotonic behavior and will be modeled with a GPI model. Fig. 7.3(c) shows the non-monotonic hysteresis relationship between the deflection output and the input current of the VO_2 -integrated microactuator, which will be modeled with an EGPI model. The total deflection range is $[48.13, 72.15]$ μm .

The deflection and resistance values were measured simultaneously, and Fig. 7.4(a) shows the hysteretic relationship between the deflection and the resistance of the microactuator. Fig. 7.4(b) shows the resistance input, which follows a pattern of damped oscillations. Closer examination (shown in Fig. 7.4(c)) of the hysteresis curve reveals a subtle behavior where the hysteresis loops do not demonstrate a strict “nested” nature under the damped oscillations of the resistance. For example, branches 1 and 2 form a major hysteresis loop, while the minor hysteresis loop formed by branches 3 and 4 is only partially inside of the major hysteresis loop formed by branches 1 and 2.

2. It can be shown that such non-nested hysteresis cannot be captured by a typical single hysteresis model (e.g., a Preisach operator or a GPI model with non-negative weighting functions).

Fig. 7.5 shows the hysteresis loops between the deflection and the current, and between the deflection and the resistance, at different frequencies of the input current. One can see that the shape of the hysteresis loop between the deflection and the current changes dramatically with the frequency, while the hysteresis loop between the deflection and the resistance has much less variation with frequency. This indicates the promise of using resistance to achieve self-sensing of deflection.

7.2 Proposed Composite Model for Self-sensing

7.2.1 Main Idea

We use hysteresis models identified under a quasi-static condition (Fig. 7.3) to derive a self-sensing model that is applicable under dynamic conditions. The justification for such an approach is as follows. Note that the phase transition in VO_2 (including both the mechanical property change and the electrical property change) is induced solely by the temperature change. And the phase change dynamics is very fast, at the order of nanoseconds [93]. Therefore, the hysteresis between the resistance and the temperature can be considered rate-independent for the frequency range of interest in this work. Similarly, the hysteresis between the deflection output and the temperature is rate-independent, within the frequency range where the structural dynamics of the cantilever is not excited. Consequently, within that same frequency range, the hysteresis between the deflection output D and the resistance Z is rate-independent, which is a key point behind our proposed approach. We note that the D - Z hysteresis in Fig. 7.5(b) shows mild rate-dependency, which can be largely attributed to the structural dynamics of the cantilever, which cannot be entirely ignored at

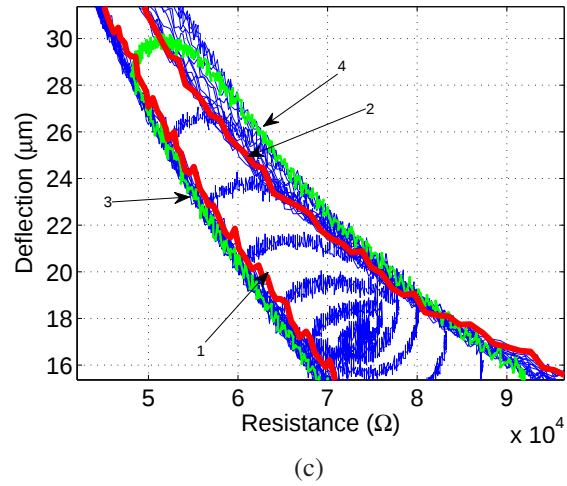
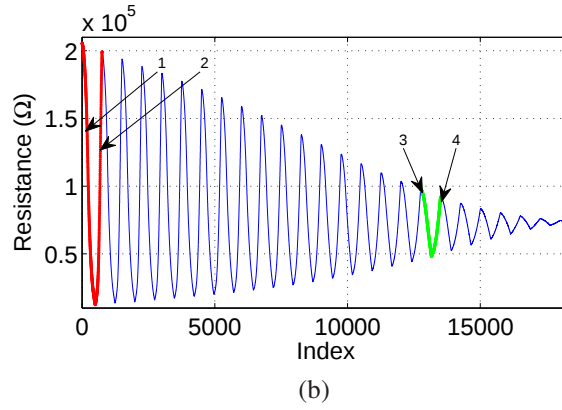
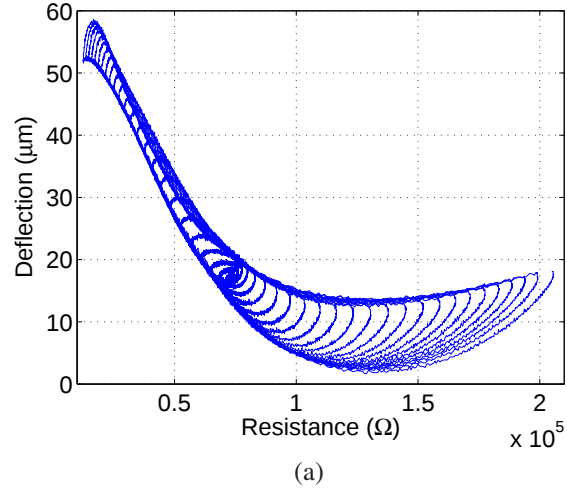
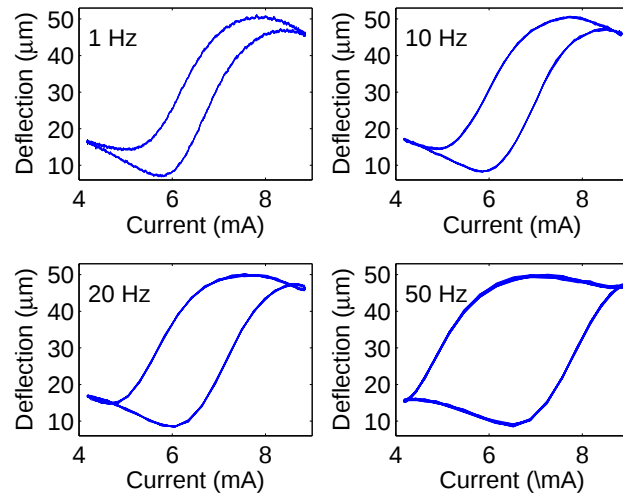
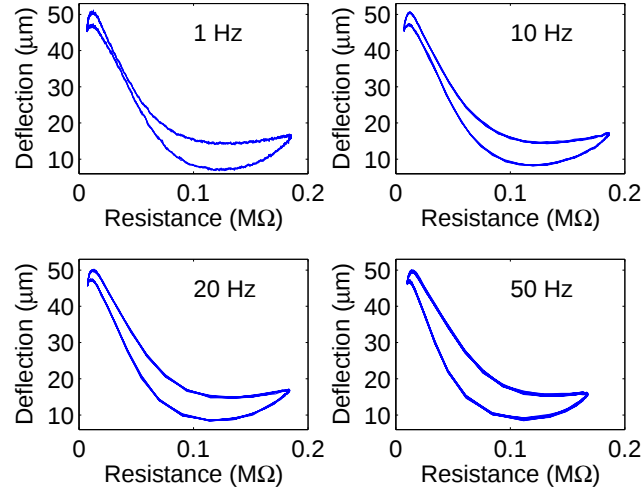


Figure 7.4: (a) The hysteresis between the deflection and the resistance; (b) the resistance sequence; (c) zoom-in plot of the hysteresis between the deflection and the resistance, revealing a non-nested structure.



(a)



(b)

Figure 7.5: The hysteresis between the deflection and the current under varying input frequencies;
(b) The hysteresis between the deflection and the resistance under varying input frequencies.

the tested frequencies.

Another key idea in the proposed method is the notion of *temperature surrogate*, which is a single-valued, strictly increasing function of the temperature. The purpose of applying quasi-static current inputs during model identification is to achieve the steady-state temperature for each value of current input, so that the relationships between resistance/deflection and temperature can be established. Since direct measurement of VO₂ temperature (which would require a dedicated sensor) is not available, the applied quasi-static current input value, i , becomes a surrogate for the steady-state temperature T as i is a single-valued, increasing function of T , namely, $i = g(T)$. Since the hysteresis between deflection D and T and the hysteresis between resistance Z and T (when structural dynamics of the cantilever is not excited) are rate-independent, so are the hysteresis between D and $g(T)$ and the hysteresis between Z and $g(T)$. This is why, *even under dynamic conditions*, we can infer the surrogate temperature $g(T)$ from the measurement Z , and then use $g(T)$ to calculate D .

Even though the explicit expression $g(T)$ for the temperature surrogate is not required for the implementation of the proposed self-sensing algorithm, for illustration purposes, we provide one example based on a simple thermal model of Joule heating [94]:

$$\frac{dT(t)}{dt} = -d_1(T(t) - T_0) + d_2 i^2(t), \quad (7.1)$$

where d_1 and d_2 are positive constants related to the density, volume, specific heat, heat transfer coefficient, resistance, and surface area of the VO₂ microactuator, and T_0 is the ambient temperature.

For a constant current i , the steady-state temperature T under (1) can be computed as $\frac{d_2}{d_1} i^2(t) + T_0$, which implies

$$i = \sqrt{\frac{d_1}{d_2}(T - T_0)} = g(T). \quad (7.2)$$

Note that the function $g(T)$ in (7.2) is indeed single-valued and strictly increasing, and thus is a legitimate surrogate for T . This notion of temperature surrogate is at the heart of our proposed self-sensing scheme. It is found in finite-element simulation with COMSOL that the thermal distribution is approximately uniform for the majority part of the cantilever, so treating the quasi-static current as a surrogate of the temperature is acceptable.

In the proposed self-sensing scheme, the deflection feedback is estimated based on the resistance measurement in two steps: first, the temperature surrogate $g(T)$ is obtained from the resistance measurement based on an inverse GPI model; second, the deflection estimate is obtained from the temperature surrogate $g(T)$ based on an EGPI model. A brief review of the GPI model and the EGPI model is provided below, and the reader is referred to [13, 20, 95] for more details on this subject.

7.2.2 Temperature Surrogate $g(T)$ based on a GPI Model

In order to capture the asymmetric hysteresis behavior between temperature surrogate $g(T)$ and the resistance output Z , a GPI model is adopted.

$$Z(t) = \sum_{j=0}^{N_1} p_1(r_j) F_{r_j}^{\gamma_1}[g(T)](t) + c_1, \quad (7.3)$$

where c_1 denotes the bias. $g(T)$ can be expressed as the following inversion model

$$g(T) = \begin{cases} \gamma_R^{-1} \circ \Pi^{-1} \circ (Z(t) - c_1), & \text{if } Z(t) > Z(t^-) \\ \gamma_L^{-1} \circ \Pi^{-1} \circ (Z(t) - c_1), & \text{if } Z(t) < Z(t^-), \\ \hat{i}(t^-), & \text{if } Z(t) = Z(t^-) \end{cases} \quad (7.4)$$

7.2.3 Estimated Deflection $\hat{D}$ Based on an EGPI Model

An EGPI model is adopted to capture the non-monotonic hysteresis behavior between the temperature surrogate $g(T)$, which can be calculated in the previous subsection, and the deflection output D :

$$D(t) = \sum_{j=0}^{N_2} p_2(r_j) F_{r_j}^{\gamma_2} [g(T)](t) + c_2 g(T) + c_3, \quad (7.5)$$

where c_2 is a constant related to the thermal expansion coefficients of the microactuator structure, and c_3 denotes a constant bias. The form of the model is chosen based on [95]. In [95], the non-monotonic hysteresis between temperature and deflection a VO_2 microactuator was modeled by the summation of a GPI model and a memoryless function.

Note that in actual operations of the actuator, the current input is not quasi-static in general and does not have a fixed relationship with the temperature. Therefore, even though the current input is readily available (as a control signal), one cannot simply use the known current value to estimate the deflection. However, the joint use of Eq. (7.4) and Eq. (7.5) will be able to produce the deflection estimate even under dynamic conditions, since the scheme operates by first estimating the temperature state $g(T)$. The discussion is also supported by comparing the hysteresis loops of the deflection and the current, and that of the deflection and the resistance, where the rate-dependency is much milder (Fig. 7.5(b)).

7.3 Model Identification and Verification

7.3.1 Model Identification

To effectively identify the model parameters, the input needs to provide sufficient excitation for individual elements of the hysteresis models. In this work an input with the form of damped oscillations is used, which produces nested hysteresis loops for the resistance-current and deflection-current relationships. For comparison purposes, an EGPI model, a single Preisach operator, and a high-order polynomial model are adopted to directly model the relationship between the deflection and the resistance. The performance of each self-sensing scheme is measured by the average and maximum absolute prediction errors. The calculation complexity is also examined using the average time of each self-sensing calculation.

The numbers of the generalized play operators in the GPI model N_1+1 and in the EGPI model N_2+1 are both chosen to be 6, the radii are chosen as $r = (i-1)/6, i = 1, 2, \dots, 6$. The numbers of play operators of the GPI model and the EGPI model are chosen such that the identified model could provide adequate accuracy with reasonable computation time. When the number of play operators is chosen to be 6, the average modeling error is less than $1 \mu\text{m}$, over the total deflection range $[1.62, 58.65] \mu\text{m}$. Increasing the number further does not seem to produce appreciable improvement in modeling accuracy. The parameters of the GPI model include play radii, envelope functions, and weights. When the number of plays is larger, the number of model parameters also is larger, posing difficulties in model identification. In practice, it is common to pre-define some of the parameters and the identified model could still accurately capture the hysteresis behavior. For example, in Chapter 6, and [13], play radii were pre-defined in a similar way as adopted in this chapter. In order to capture the hysteresis between the temperature surrogate and the resistance, the envelope functions for the generalized play operator are chosen to be hyperbolic-tangent functions

Table 7.1: Identified parameters of the GPI model.

c_1	-132556.3
a_L	519.7
a_R	624.9
b_L	-2.984
b_R	-3.656
p_1	(0.415, 0.648, 0.325, 0, 0.081, 0.618)

in the form of:

$$\gamma_R(v(t)) = \tanh(a_R v(t) + b_R), \quad (7.6)$$

$$\gamma_L(v(t)) = \tanh(a_L v(t) + b_L). \quad (7.7)$$

Hyperbolic-tangent functions could effectively capture the complicated asymmetric hysteresis with output saturation in VO₂ microactuators. The hyperbolic-tangent functions have also been adopted to model other types of hysteresis behaviors and [13] and their effectiveness have been verified.

The model parameters are identified through minimization of an error-squared function between the actual deflection and the model using the Matlab optimization toolbox [13]. Table 7.1 and Table 7.2 show the parameters of the GPI model and the EGPI model, respectively. Note that for the GPI model, all the generalized play operators have the same envelope functions.

Fig. 7.7(a) shows the performance of the proposed self-sensing scheme, and Fig. 7.7(b) shows the prediction error. The average and maximum absolute errors with the composite model are 0.95 μm and 4.01 μm , respectively, over the total deflection range [1.62, 58.65] μm . The average time for each self-sensing calculation is 0.16 ms. The computations were run in Matlab on a computer Lenovo Thinkpad T420 with 2.80 GHz CPU and 4.00 GB memory.

Table 7.2: Identified parameters of the EGPI model.

c_2	-8874.3
c_3	143.29
a_L	(982.5, 1754.8, 2169.2, 838.1, 1864.7, 1623.2)
a_R	(1202.6, 1376.2, 1243.0, 1173.7, 788.8, 788.8)
b_L	(-6.49, -11.13, -11.56, -6.24, -6.46, -15.52)
b_R	(-8.67, -12.15, -10.12, -7.42, -10.83, -11.21)
p_2	(27.24, 5.35, 3.18, 12.84, 1.00, 26.68)

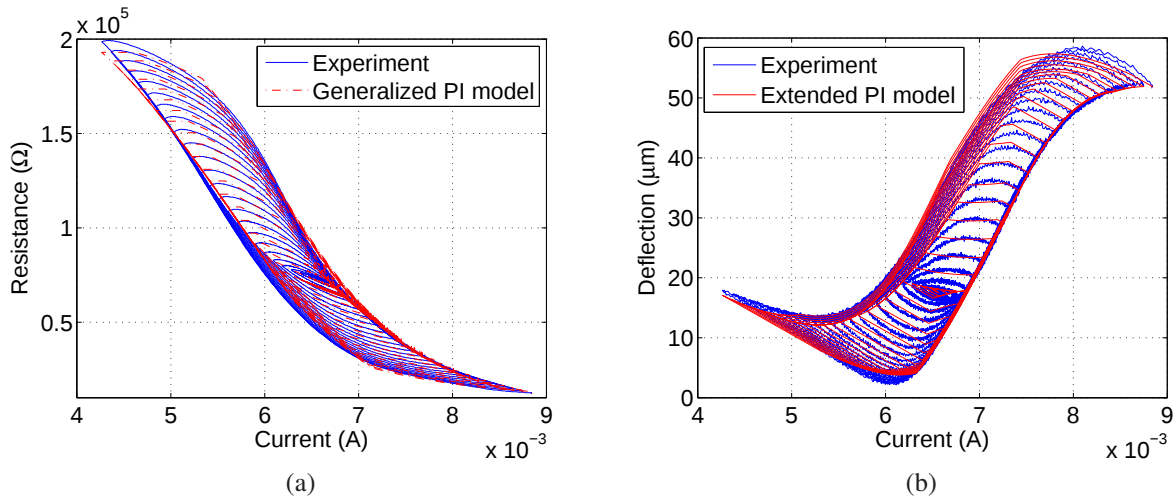


Figure 7.6: (a) The comparison between the GPI model prediction and experimental measurement for the asymmetric hysteresis between the resistance output and the current input; (b) the comparison between the EGPI model prediction and experimental measurement for the non-monotonic hysteresis between the deflection output and the current input.

Fig. 7.7(a) shows the performance of the proposed self-sensing scheme, and Fig. 7.7(b) shows the prediction error. The average and maximum absolute errors with the composite model are $0.95 \mu\text{m}$ and $4.01 \mu\text{m}$, respectively, over the total deflection range $[1.62, 58.65] \mu\text{m}$. The average time for each self-sensing calculation is 0.16 ms . The computations were run in Matlab on a computer Lenovo Thinkpad T420 with 2.80 GHz CPU and 4.00 GB memory.

The Preisach model is a popular and effective hysteresis model [9, 12, 85]. A Preisach model consists of weighted superposition of delayed relays. Practical parameter identification involves

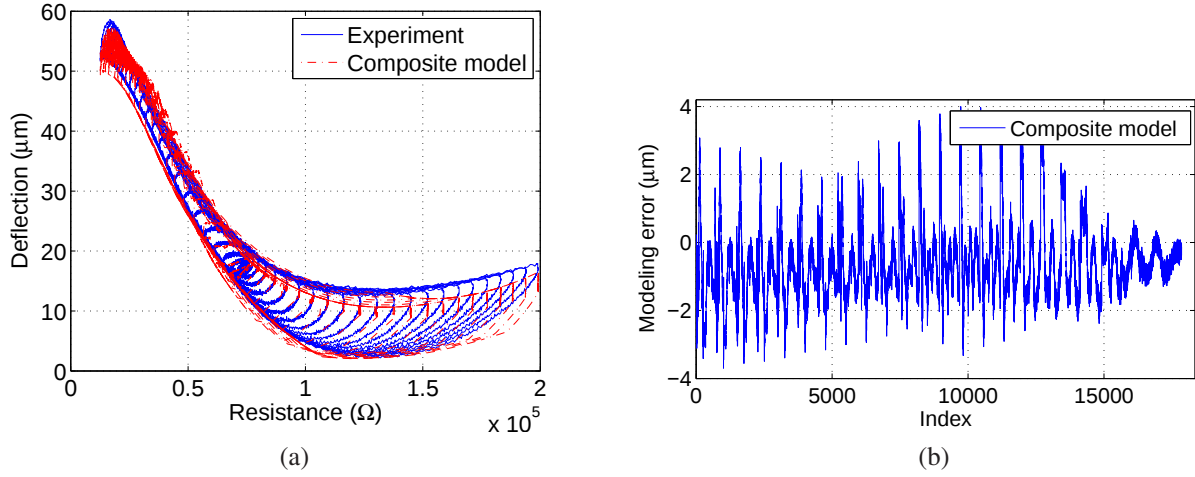


Figure 7.7: (a) Performance of the self-sensing scheme using the composite model; (b) the self-sensing error based on the composite model.

discretization of the Preisach density function in one way or another, and one effective method is to approximate the density with a piecewise constant function [9]. A non-monotonic hysteresis model that combines a monotonic Preisach model with a memoryless operator is adopted to directly model the hysteresis between resistance and deflection. The number of discretization level of the model is chosen to be 10. Fig. 7.8(a) shows the modeling performance of the Preisach model. The average and maximum absolute errors are $1.19 \mu\text{m}$ and $6.68 \mu\text{m}$, respectively. The average time needed for each self-sensing calculation is 0.68 ms. Therefore, the Preisach model results in much longer calculation time and producing less accurate modeling performance as compared to the proposed approach. Moreover, the modeling performance shows that the Preisach model cannot capture the non-nested hysteresis loops.

An EGPI model that combines a GPI model with a memoryless operator is also adopted for modeling comparison. The number of generalized play operators of the model is chosen to be 6. Fig. 7.8(b) shows the modeling performance of the EGPI model. The average and maximum absolute errors are $1.26 \mu\text{m}$ and $4.65 \mu\text{m}$, respectively. The average time needed for each self-sensing calculation is 0.11 ms. Therefore, the EGPI model results in comparable calculation time but pro-

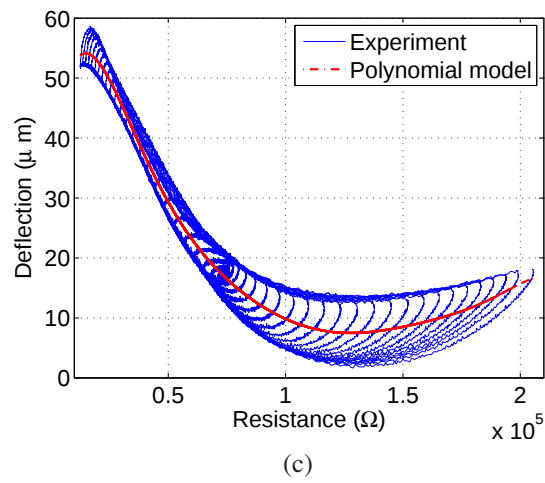
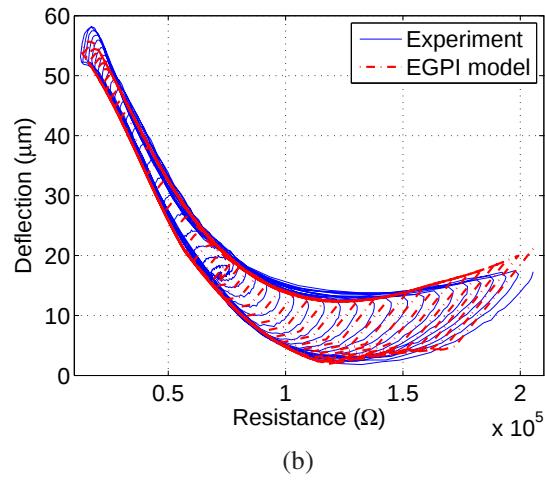
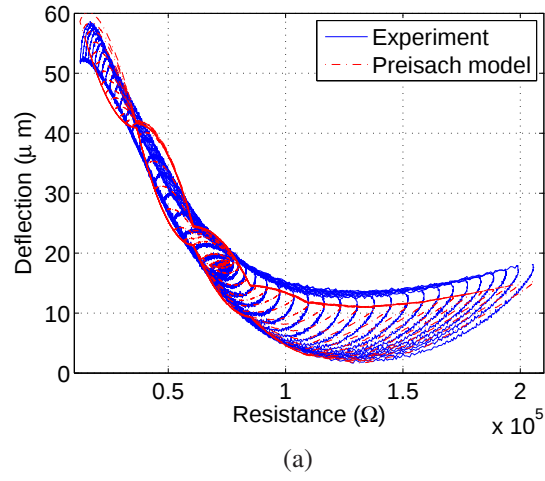


Figure 7.8: Performances of the self-sensing schemes using (a) a Preisach model; (b) an EGPI model; (c) a high-order polynomial model.

duces less accurate modeling performance as compared to the proposed approach. Moreover, the modeling performance shows that the EGPI model cannot capture the non-nested hysteresis loops. Due to the similar modeling accuracy as the Preisach model, the EGPI model is not adopted for model verification or control experiments.

A eighth-order polynomial model is identified to approximate the complicated hysteresis relationship between the deflection and the resistance, as shown in Fig. 7.8(c). It can be seen that a polynomial model fails to capture the hysteresis relationship, and the average and maximum absolute errors in self-sensing are $2.66 \mu\text{m}$ and $6.48 \mu\text{m}$, respectively, while the average time needed for each self-sensing estimation is only 0.004 ms. Although the polynomial model takes much less time than the composite model, its modeling performance is much worse than the proposed approach.

We have shown that, for some chosen designs, the proposed composite hysteresis model-based self-sensing scheme outperforms the Preisach model-based schemes in both precision and efficiency, and outperforms the EGPI and polynomial model-based schemes in precision with higher computational complexity. On the other hand, it is known that the error performance of each scheme depends on the complexity of each model. Here a more in-depth comparison is provided by varying the complexity of each scheme. Fig. 7.9 compares the self-sensing performance and computational time of each model when the “level” of each is varied from 6 to 10. Here the term “level” refers to the number of generalized play operators for the GPI model and the EGPI model, the discretization level for the Preisach model, and the degree of the polynomial model, respectively. It can be seen that the composite model consistently has the lowest modeling error among the four schemes. Furthermore, its computational complexity is only slightly higher than that of the EGPI model-based schemes. Additionally, unlike other schemes, the proposed model can capture the subtle deflection-resistance hysteresis behavior where the hysteresis loops do not demonstrate

a strict “nested” nature under the damped oscillations of the resistance.

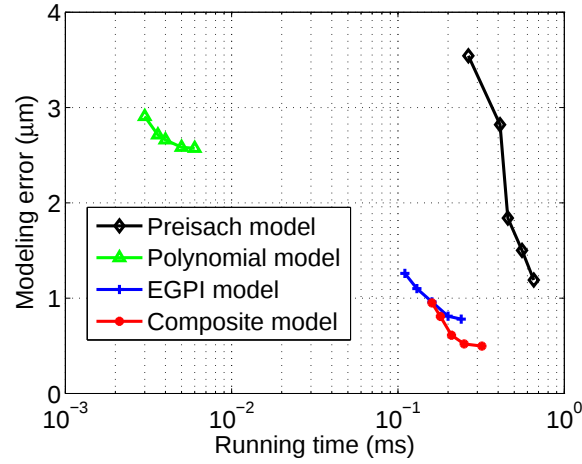
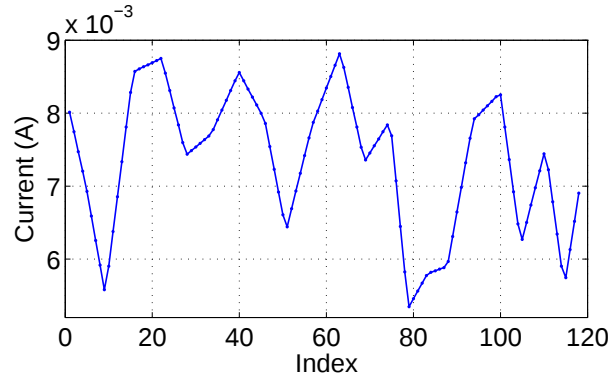


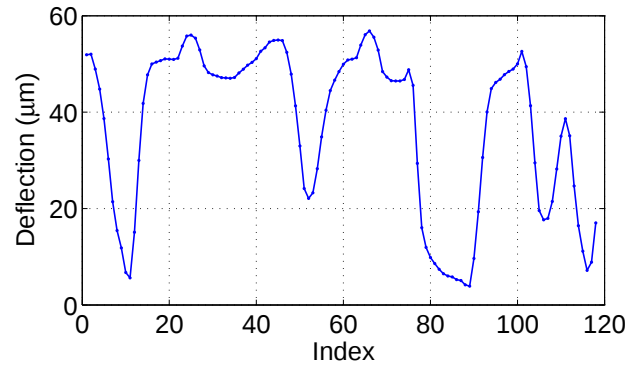
Figure 7.9: Model accuracy and the running time comparison between the composite model, the Preisach model, the EGPI model, and the polynomial model.

7.3.2 Model Verification

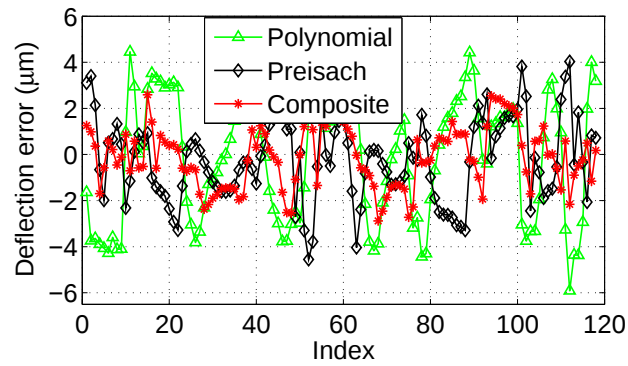
In order to further validate the proposed approach, the VO₂-integrated microactuator is subjected to a randomly chosen current input sequence, shown in Fig. 7.10(a), under each of the three schemes. For each index, the current is held for 10 ms. Here the numbers of generalized play operators in the GPI model and the EGPI model for the proposed scheme is 6, the discretization level for the Preisach model is 10, and the order of the polynomial model is 8. Fig. 7.10(b) shows the experimental measurement of the deflection and Fig. 7.10(c) shows the self-sensing errors under each scheme. The average absolute errors are 1.10 μm , 1.45 μm , and 2.35 μm , respectively, under the composite model, the Preisach model and the polynomial model. The maximum absolute errors are 2.88 μm , 4.55 μm , and 5.91 μm , respectively, under the composite model, the Preisach model and the polynomial model. The effectiveness of the proposed model is thus further verified.



(a)



(b)



(c)

Figure 7.10: (a) A randomly chosen current input sequence for self-sensing model verification; (b) the experimental deflection measurement under the random current input sequence; (c) errors in predictions by different self-sensing approaches.

7.4 Self-sensing-based Feedback Control

The block diagram for the physical closed-loop system is shown in Fig. 7.11. The input of the controller is the deflection error between the reference and the self-sensed deflection. The output of the controller is the current. The heating dynamics is modeled as a first-order system and the time constant is identified to be 1.8 ms based on a series of step response experiments. Proportional-integral tuning is conducted in simulation. The following proportional-integral parameters was chosen to ensure desirable step response and fast sinusoidal-tracking performance: $K_p = 1.4 \times 10^{-3}$, $K_I = 2.03 \times 10^{-5}$.

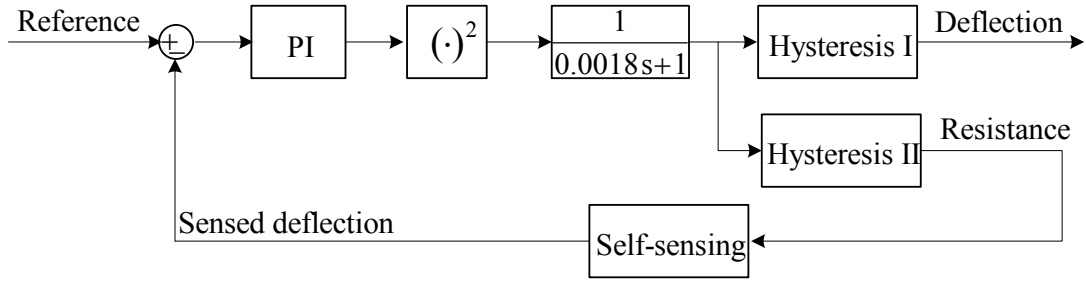


Figure 7.11: Block diagram of the closed-loop control system with self-sensing.

7.4.1 Step Reference Tracking

A step reference-tracking experiment has been first conducted. Each reference setpoint has duration of 1 s. Fig. 7.12 shows the experimental performance in terms of the reference and the actual deflection measured by the external PSD. Note that although the self-sensed deflection is used in the feedback control, the actual deflection is of more relevance. The average absolute tracking errors under the composite model, the Preisach model, and the polynomial are $1.11 \mu\text{m}$, $2.21 \mu\text{m}$, and $3.12 \mu\text{m}$, respectively. The step reference-tracking experiment demonstrates the effectiveness of the proposed composite self-sensing model.

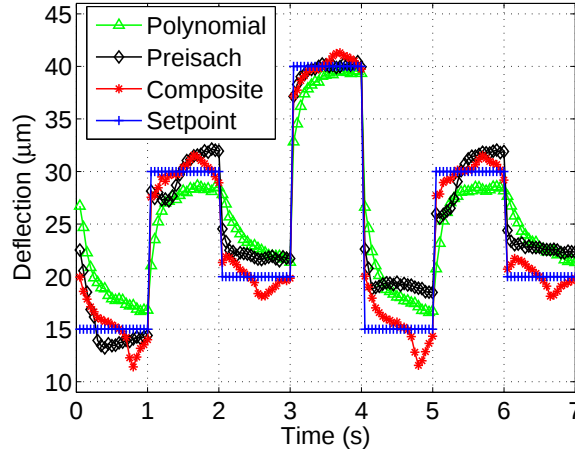


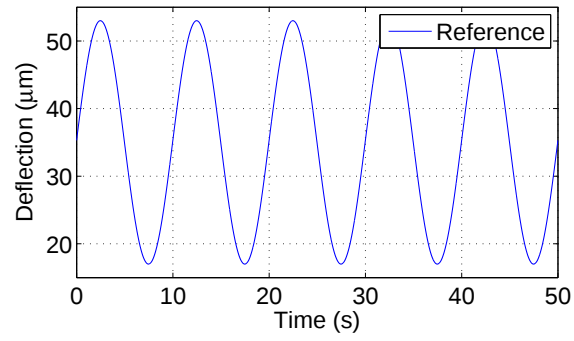
Figure 7.12: Experimental performance of tracking a step reference under different self-sensing schemes.

7.4.2 Sinusoidal Reference Tracking

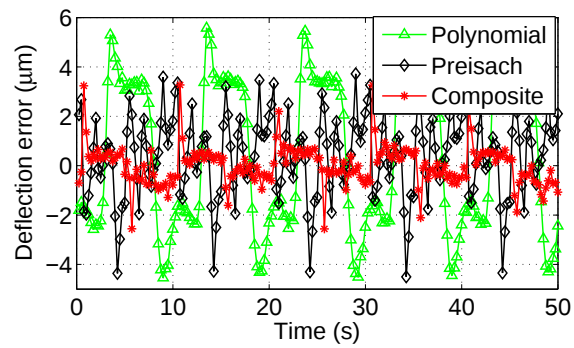
The next experiment involves the tracking of a sinusoidal signal. The reference signal, shown in Fig. 7.13(a), has frequency of 0.1 Hz. Fig. 7.13(b) shows the tracking performance of the three self-sensing approaches. It is calculated that the controllers based on the composite self-sensing approach, the Preisach model, and the polynomial model result in average absolute errors of $0.69 \mu\text{m}$, $1.28 \mu\text{m}$, and $2.79 \mu\text{m}$, respectively. The results show the effectiveness of the proposed self-sensing approach for feedback control.

7.4.3 Multi-frequency Reference Tracking

Experiments on tracking multi-frequency signals have been further conducted. The reference signal is chosen as $4\sin(2\pi t) - 6\sin(2\pi 10t) + 30$, which is shown in Fig. 7.14(a). Fig. 7.14(b) shows the tracking performance of the three self-sensing approaches. It is calculated that the controller based on the composite self-sensing approach results in an average absolute error of $2.43 \mu\text{m}$, which is 20.1% and 44.1% less than those under the Preisach model and the polynomial model-based schemes, respectively. The results show that the controllers result in larger tracking error



(a)



(b)

Figure 7.13: (a) A sinusoidal reference signal for tracking control of the VO_2 -integrated microactuator; (b) experimental tracking errors under different self-sensing schemes.

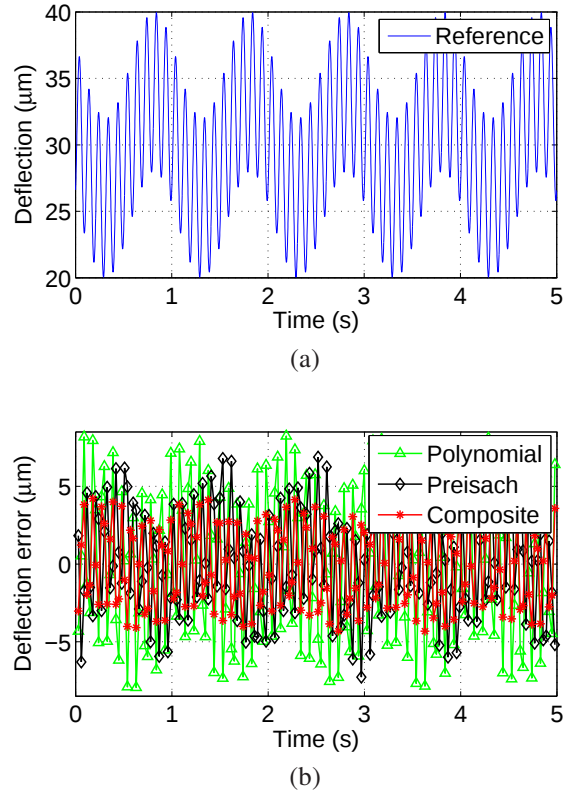


Figure 7.14: (a) A multi-frequency reference signal for tracking control of the VO_2 -integrated microactuator; (b) experimental tracking errors under different self-sensing schemes.

comparing with the tracking of a lower-frequency signal (Fig. 7.13), which is likely due to the mild frequency-dependence of the deflection-resistance hysteresis (Fig. 7.5) that is not captured in these models.

Chapter 8

Robust Control of VO₂ Microactuators using Self-Sensing Feedback

In this chapter, self-sensing-based robust control of VO₂ microactuators is studied. Although the resistance change is due to an insulator-to-metal-transition (IMT) and the mechanical change is due to a structural-phase-transition (SPT), these two mechanisms are strongly coupled. Thus, self-sensing is achieved by mapping deflection to resistance with a high-order polynomial. By employing this technique, not only the impact of hysteresis can be reduced due to the highly coupled deflection and resistance changes in VO₂, but the measurement setup is also greatly simplified. The robust controller takes into account the error in modeling temperature-deflection hysteresis, and environmental disturbances. The controller takes into consideration the error between the desired and actual deflection values in order to precisely control the microactuator. The performance of the robust controller is also compared to a PID controller.

8.1 Experimental Procedures

8.1.1 VO₂ Deposition

The VO₂ thin film was deposited, through pulse laser deposition, on a chip containing a Si micro-cantilever with length, width, and thickness of 300, 35, and 1 μm , respectively. The microactuator

chip was attached to a Si test piece and was placed in a vacuum chamber with a mixed gas pressure of argon (40 %) and oxygen (60 %) at 20 mTorr and maintained through a 30 min deposition. A ceramic heater, controlled at 600 °C, was used to heat the sample during deposition. Although the temperature at the sample was not directly measured, a calibration done before the deposition approximates the temperature at 550 °C. The sample was also rotated throughout the deposition to ensure uniform temperature and thickness distribution. A krypton fluoride excimer laser was focused on a rotating vanadium target 5 cm apart from the sample with an intensity of 350 mJ and a repetition rate of 10 Hz. After deposition, the VO₂ thickness was measured to be 172 nm. To determine the quality of the VO₂, the resistance of the film on top of the test chip was measured as a function of temperature through a heating-cooling cycle (20-85 °C) (vide infra). A drop of two orders of magnitude in film resistance is observed, which is similar to resistance changes reported in the literature for stoichiometric polycrystalline VO₂ on Si substrates [96].

8.1.2 Measurement Setup

The measurement setup is similar as Fig. 5.1. The VO₂-coated Si microactuator (shown in cross-sectional view) was attached to the same test piece used during deposition, which was also Si coated with the same VO₂. This test piece was needed in order to create the electrical connections to the VO₂ and measure its resistance. These contacts were located next to the microactuator chip and fabricated by evaporating aluminum through a custom-made metal mask. A voltage divisor (not shown in the schematic) was used in order to measure the resistance of the VO₂ film.

To measure the deflection of the device, a sensing laser ($\lambda = 808$ nm, 0.5 mW) was focused on the tip of the microactuator and the reflected light was then focused on the active area of a PSD. A charged-coupled device camera was used to aid in the alignment of the laser. The PSD output was a voltage proportional to the deflection of the microactuator, which was calibrated by

sideview images of the cantilever at different deflection values. In particular, the calibration of the PSD reading was done by first assigning the initial deflection of the cantilever at 20 °C as 0 μm . Then the sample was heated to 85 °C, which resulted in the maximum deflection of the cantilever, 70 μm , as measured from the side view images of the cantilever. The total voltage change in the PSD output from 20 to 85 °C was 7.8 V, resulting in a measurement sensitivity of 0.111 V/ μm . We note that the PSD used in this work had a resolution of 0.2 μm for the laser spot displacement, while the range for the laser spot displacement was 13.875 mm when the temperature varied from 20 to 85 °C. This means that the PSD reading was highly accurate since the output resolution was 1.4×10^{-5} of the operational range. A Peltier heater was used to control the temperature of the sample. The temperature at the heater was measured with a platinum temperature sensor. A data acquisition card and field programmable gate array (DAQ/FPGA) was used to access the PSD output, the resistance of the VO₂ film, and the temperature sensor output. The DAQ/FPGA system was programmed to either: 1) control the temperature of the Peltier in closed loop in order to measure the deflection of the microactuator and the VO₂ resistance of the test piece simultaneously, or 2) control, using PID or robust controller, the deflection of the microactuator by self-sensing the deflection through resistance. For both cases, the DAQ/FPGA controlled the magnitude of the current signal sent to the Peltier heater. All the variables were controlled and observed in a computer connected with the DAQ/FPGA system.

8.2 Self-Sensing Deflection

Fig. 8.1 shows the major heating-cooling cycle of the microactuator deflection and film resistance as a function of temperature. The deflection in this work is defined as the tip displacement change relative to the initial position. A total deflection of 70 μm and a resistance drop of two orders

of magnitude were measured during the VO_2 transition through a temperature span of 15°C . Both variables were simultaneously measured, and by mapping deflection with resistance, it was observed that the hysteresis between the deflection and the resistance was insignificant, enabling the use of the resistance of the film in the test piece to estimate the deflection without the need of physically measuring its value.

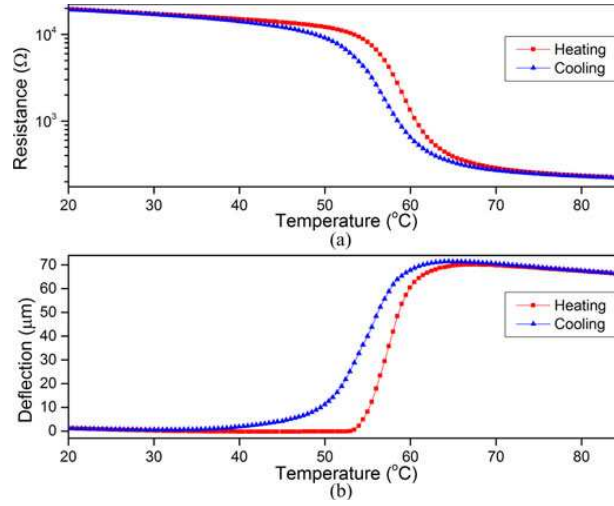


Figure 8.1: (a) VO_2 film resistance, and (b) VO_2 -coated microactuator actual deflection as a function of temperature through a heating-cooling cycle ($20\text{--}85^\circ\text{C}$). Both variables were simultaneously measured.

The deflection-resistance mapping is shown in Fig. 8.2(a), which also includes a ninth-degree polynomial used to estimate the deflection in the experiments. This model was obtained from fitting the average of the heating and cooling curves and was used as the deflection sensing mechanism in the closed-loop deflection control experiments done in this work. The maximum errors between the heating/cooling curves and the self-sensing model are shown in Fig. 8.2(b). For a wide range of the resistance, the deflection estimation error was lower than $2\ \mu\text{m}$ whereas slightly larger estimation error was found at the two ends. It is observed that some hysteresis remains. This is believed to be due to the slightly different energy requirements between the IMT and the SPT [87]. This hypothesis is supported by the fact that this difference in energy requirements has been found

to be more pronounced at the onset of the phase transition, which would correspond to the higher resistance-low deflection region in Fig. 8.2(a). Hereinafter, the estimated and measured deflection values will be addressed as self-sensed and actual deflections, respectively.

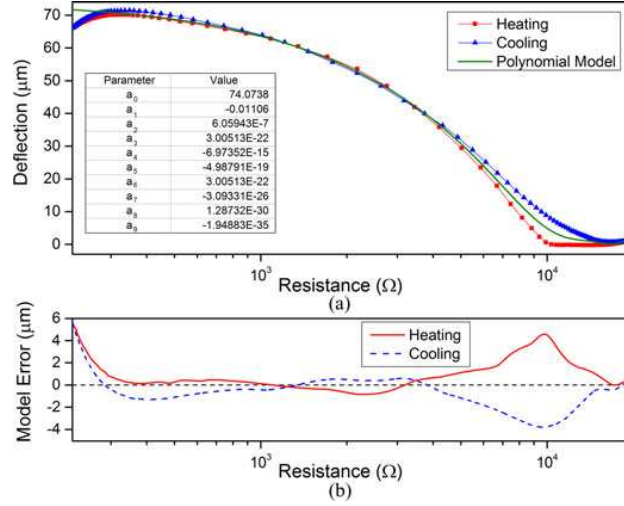


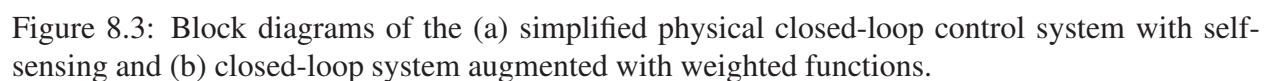
Figure 8.2: (a) VO₂-coated microactuator actual deflection as a function of VO₂ film resistance during the heating-cooling cycle. A polynomial function of degree 9 was used to model the deflection-resistance mapping. (b) Maximum model error obtained from the major heating and cooling curves.

Two types of controllers will be considered and compared: 1) a PID controller, which only considers the error between the controlled variable (in this case self-sensed deflection) with the desired reference signal, and 2) a robust controller, which aside from considering the error from the controlled variable, also accommodates the error brought by the self-sensing model, noises, and system uncertainties.

8.3 Robust Controller Design

Following H_∞ design techniques [97], an H_∞ controller was designed to accommodate perturbations, noises, and model uncertainties in deflection tracking.

The block diagram for the simplified physical closed-loop system with self-sensing is shown in Fig. 8.3(a) and the modeled closed-loop system augmented with weighted uncertainties is shown in Fig. 8.3(b). The variable y_{ref} is defined as the desired deflection output. The input of controller $K(s)$ is the deflection error defined as the difference between y_{ref} and the self-sensed deflection y_{self} . The controller output is the current I_c , which is corrupted by I_d that account for environmental disturbances d . $A(s)$ denotes the transfer function for the Peltier heater with temperature T as its output. The hysteresis between temperature T and actual deflection of the VO₂ cantilever y is modeled by the summation of a linear relationship K_c and a noise n_1 . The VO₂ film resistance is defined as R . The self-sensing error is taken into consideration through a noise n_2 . The functions W_u , W_d , W_{n_1} , W_{n_2} , and W_e are weighting functions that represent the importance of the corresponding signals. For example, W_u reflects the control effort constraints and W_e accounts for actual deflection performance. The variables $\tilde{u}$ and $\tilde{e}$ are weighted input and weighted deflection error, respectively.



The temperature dynamics due to the Peltier heater are modeled by a first-order system and a time delay represented by a first-order Padé approximation [98]

$$A(s) = \left(\frac{-\frac{\tau_d}{2}s + 1}{\frac{\tau_d}{2}s + 1} \right) \left(\frac{-A_0}{\tau s + 1} \right) \quad (8.1)$$

where τ is the time constant associated with the system transient, A_0 is the gain, and τ_d is the time constant associated with the system delay. In this work, the deflection transfer function was assumed to have no dynamics since the time constants associated with heat transfer through the cantilever and drag produced by air are much lower than that of the Peltier heater dynamics. Hence, the Peltier heater dynamic response was considered the dominant dynamic in the system under study.

A series of open-loop step input experiments were conducted to identify the system parameters and the results are shown in Table 8.1. These were done by manually controlling the current through the Peltier and measuring the temperature transients. The initial current value for each experiment was zero, which corresponded to 25 °C (room temperature). Only heating steps were considered in this parameter identification and a parameter δ was used as an uncertainty parameter due to the differences from heating and cooling with $\delta \in [-1, 1]$. From the measured data, the time constant τ and the gain A_0 of the plant model were calculated to be $25(1 + 0.2\delta)$ and 50 °C/A, respectively. The 0.2 factor that multiplies δ is chosen to cover the range of measured time constants, which span from 20 to 30 s. The time delay τ_d , which is defined here as the time interval between a change in the input current to the system and the temperature response to that signal (dead time), was experimentally measured to be 0.375 s.

There exists a considerable amount of hysteretic nonlinearity between temperature and deflection of the VO₂ microactuator. Although there are several models that capture the hysteresis

Table 8.1: Steady-state values of step experiments for system identification.

<i>Current</i> I (A)	Temperature T ($^{\circ}\text{C}$)	Time constant τ (s)
-0.2 (heating)	39.7	20.8
-0.4 (cooling)	49.2	27.4
-0.6 (heating)	59.1	22.6
-0.8 (cooling)	70.1	28.2

memory effects in microactuators, due to the complexity consideration in online processing, the hysteresis nonlinearity in this work was approximated as follows:

$$y = K_e T + n_1 W_{n_1} \quad (8.2)$$

where K_e is the rate of change in deflection as a function of temperature across the transition, which was identified to be $5.9 \mu\text{m}/^{\circ}\text{C}$. The second term in the sum represents the hysteresis modeling error. The self-sensed deflection was then modeled by

$$y_{self} = K_e T + n_1 W_{n_1} + n_2 W W_{n_2} \quad (8.3)$$

where the third term in the sum represents the self-sensing error obtained with the high-order polynomial in Fig. 8.3(a). By considering these modeling errors and the remaining weighting functions in Fig. 8.3(b), a robust control framework can be designed that accommodates environmental disturbances, modeling errors, and uncertainties while effectively controlling the actual deflection of the VO_2 -based microactuators.

8.3.2 Robust Controller Design

Fig. 8.4(a) shows the system framework after a linear fractional transformation (LFT), which facilitates the H_∞ controller design process. The transfer functions $\Delta(s)$, $P(s)$, and $K(s)$ denote the uncertainty, interconnection matrix, and controller, respectively. The interconnection matrix is denoted as follows:

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

where the lower LFT is defined as $F_l(P, \Delta_l) = P_{11} + P_{12}\Delta_l(I - P_{22}\Delta_l)^{-1}P_{21}$ and, similarly, the upper LFT as $F_u(P, \Delta_u) = P_{22} + P_{21}\Delta_u(I - P_{11}\Delta_u)^{-1}P_{12}$ with compatible dimensions. The H_∞ control design objective is to find a controller $K(s)$, such that, given $\gamma > 0$, it minimizes the H_∞ norm of the transfer function from the input $W = \begin{pmatrix} d & n_1 & n_2 & y_{ref} \end{pmatrix}^\top$, which includes the noises, disturbance, and reference signal, to the output $z = \begin{pmatrix} \tilde{e} & \tilde{u} \end{pmatrix}^\top$, which includes the control effort and tracking error, by solving

$$\|F_l(F_u(P(s), \mathbf{0}), K(s))\|_\infty < \gamma \quad (8.4)$$

where $F_u(P(s), \mathbf{0})$ represents the nominal model and $\|\cdot\|$ denotes the H_∞ norm.

Choosing appropriate weights is very crucial in robust control design. The main guidelines in this work are as follows: 1) the control effort weight W_u and disturbance rejection weight W_d are very important across a wide frequency range in order to deal with disturbances with arbitrary frequencies; 2) the actual deflection performance weight W_e is also given importance, especially at low frequencies since the frequency of the desired deflection signal is relatively low due to the relative big time constant of the temperature dynamics; and 3) the noise weightings W_{n_1} and W_{n_2}

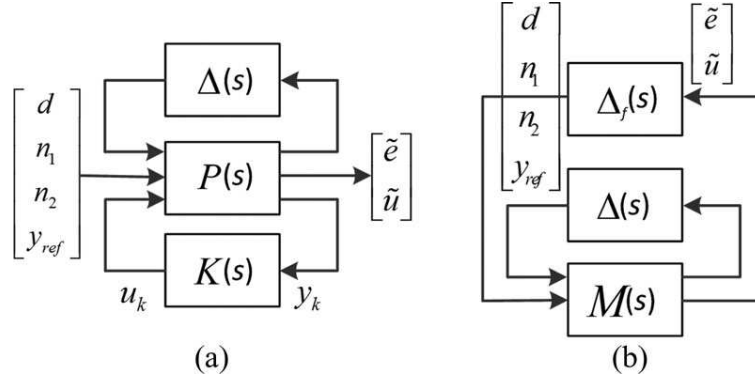


Figure 8.4: (a) Framework of H_∞ control for the system and (b) robust performance test by augmenting the uncertainty Δ to M .

are more important at higher frequencies since noises will usually have higher frequencies than those of the reference signals. With these guidelines, the transfer functions for the weights are chosen as follows:

$$W_u = 0.12 \frac{(s+1)}{\left(\frac{s}{10} + 1\right)}, \quad (8.5)$$

$$W_d = 0.1 \frac{\left(\frac{s}{2} + 1\right)}{(s+1)}, \quad (8.6)$$

$$W_e = 0.09 \frac{1}{\left(\frac{s}{10} + 1\right)}, \quad (8.7)$$

$$W_{n1} = 0.002 \frac{(s+1)}{\left(\frac{s}{100} + 1\right)}, \quad (8.8)$$

$$W_{n2} = 0.002 \frac{(s+1)}{\left(\frac{s}{100} + 1\right)}. \quad (8.9)$$

Based on the model parameters and the weighting functions for the model in Fig. 8.3(b), the system can be expressed in the following state-space representation, $P(s) = C(sI - A)^{-1}B + D$, where

$$A = \begin{bmatrix} -0.04 & 0 & 0 & 50 & 0 & 0 & -50 \\ 0 & -10 & 0 & 50 & 0 & 0 & 0 \\ -0.24 & 0 & -10 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -5.33 & 0 & 0 & 10.67 \\ 0 & 0 & 0 & 0 & -100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.008 & 0 & 0 & 2.5 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 & 10.8 \\ 0.048 & -0.2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -5.33 & 0 & -10.67 \\ 0 & 19.8 & 0 & 0 & -0 & 0 \\ 0 & 0 & 19.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.05 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -0.24 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.2 \\ -0.048 & -0.2 & -0.2 & 0 & 1 & 0 \end{bmatrix}.$$

The transfer function of the controller is calculated based on the algebraic Riccati equation

$$K(s) = \frac{-918(s+1.1)(s+5.3)(s+8.2)(s+100)}{(s+40830)(s+89)(s+1)(s+4.4 \pm 1.6i)}. \quad (8.10)$$

In order to implement the resulting controller in Eq. (8.10) it is changed to its discrete z-transform with a sampling time of 125 ms. This sampling time was more than an order of magnitude faster than the closed-loop control response of the system, which ensured a fully reconstructed signal. The variable γ_{opt} is the optimum over all γ such that the controller is admissible, and it was calculated to be 0.118.

In order to verify the robustness of the closed-loop system, there are two specifications to test: robust stability and robust performance. Black-Nichols diagram has been utilized to analyze the robust stability in [99], whereas small gain theory [100] and μ synthesis [97, 101] have been utilized to analysis robust stability and robust performance. The latter is a unified approach for analyzing robust stability and robust performance with multiple sources of uncertainties, which is advantageous over the small gain theory approach. Thus, μ synthesis is adopted in this work, which is represented in Fig. 8.4(b).

To test for system robust stability, denote

$$M(s) = F_l(P(s), K(s)) = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix},$$

if $\|M_{11}\|_\infty < \frac{1}{\beta}$ and $\beta > 0$ are always satisfied for all $\Delta(s)$ with $\|\Delta\|_\infty < \beta$, then the system is robustly stable. For the system shown in Fig. 8.3(b), it is verified that $\|M_{11}\|_\infty = 0.26$, which makes the system robustly stable since $\|\Delta\|_\infty = \|\delta\|_\infty < 1$. To test for robust performance, assume $M(s)$ have $q_1 + q_2$ inputs and $p_1 + p_2$ outputs, $M_{11}(s)$ has q_1 inputs and p_1 outputs, and denote

$$\Delta_p = \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_f \end{bmatrix} : \Delta \in \Delta, \Delta_f \in C^{q_2 \times p_2} \right\}$$

where $\Delta_p = \delta \in [-1, 1]$ in this work and Δ_f is shown in Fig. 8.4. A structured singular value can be defined as follows:

$$\mu_{\Delta_p}(M) = \frac{1}{\min[\sigma(\Delta) : \Delta \in \Delta \det(I - M\Delta) = 0]} \quad (8.11)$$

where $\sigma(\Delta)$ is the largest singular value of Δ .

If for all $\Delta(s)$ with $\|\Delta\|_\infty < \beta$ and $\beta > 0$, $\sup_{w \in R} \mu_{\Delta_p}(M(jw)) \leq 1/\beta$ is always satisfied, then the system has robust performance. Based on D-K iterations, $\sup_{w \in R} \mu_{\Delta_p}(M(jw))$ is found to be 0.32, for the case presented here. Hence, the robust performance of the designed robust controller is verified.

8.4 Experimental Result

Experimental results are provided using the H_∞ controller for step and multisinusoidal reference inputs with and without added noise to the current generator. Its performance is compared to that of a PID controller in order to show its robustness to noises and perturbations, as well as the control effort advantages over the PID controller. We have chosen a PID controller instead of a proportional-derivative controller for the comparison, because, for the dynamics shown in the

system, a PD controller would result in nonzero steady-state error for step references even under ideal conditions. The RMSE has been selected to quantify the tracking error in all the experiments, although the average steady-state error has been also calculated for experiments with step reference inputs. Standard deviation (SD) was used to measure the control effort in the multisinusoidal reference input experiment with and without noise.

The parameters of the PID were tuned in simulation based on the same nominal model shown in Fig. 8.3(b). Since most of the potential applications for the presented microbenders will require high-precision and fast response, the PID controller was designed to have an overshoot of less than 2%, which would ensure accuracy during the transients, and to have a settling time of smaller than 10 s, which would ensure relatively fast response given that the time constant of the Peltier dynamics was approximately 25 s. The resulting controller parameters were: proportional gain $K_p = 0.059$, integral gain $K_i = 0.004$, and derivative gain $K_d = 0.0136$. The obtained transfer function for the controller was transformed to its discrete counterpart in the z-transform with the sampling time of 125 ms for implementation, similar to the robust controller case.

8.4.1 Step Reference Tracking

Experiments with step reference inputs were designed so that the microactuator followed a set of three different setpoints, each with duration of 15 s, programmed in the DAQ/FPGA. The goal of these experiments was to study the transient behavior and steady-state error of the robust controller and compare those to the performance of PID controller. Fig. 8.5(a) and (b) shows the experimental performance in terms of the actual deflection and self-sensed deflection. Although the controlled variable is the self-sensed deflection and a better steady-state performance is observed in Fig. 8.5(b) for the PID, Fig. 8.5(a) shows that the actual steady-state deflection under the robust controller is closer to the setpoint for every step value, whereas it has a higher difference under the PID

controller. The actual steady-state deflection errors and control efforts are shown in Fig. 8.6(a) and (b).

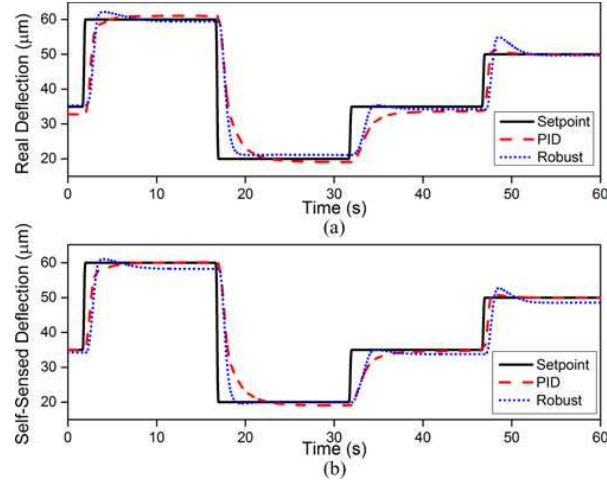


Figure 8.5: (a) Actual, and (b) self-sensed microactuator deflection under self-sensed, closed-loop PID and robust control through a series of step reference inputs.

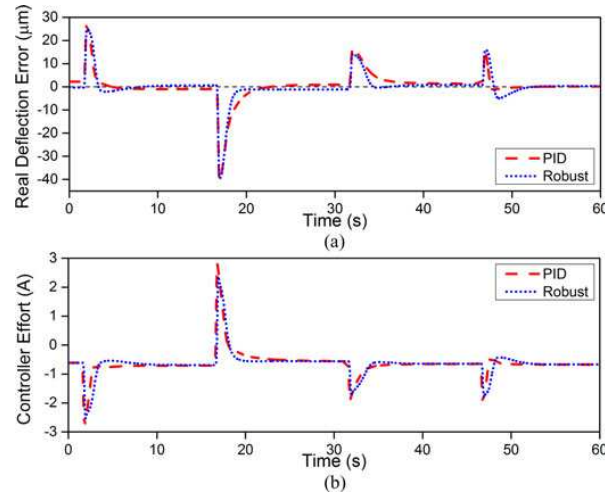


Figure 8.6: (a) Actual deflection error and (b) controller effort for the PID and robust control approaches through the step reference tracking experiment.

Table 8.2 compares the RMSE of the actual deflection and the average (over three setpoints) steady-state deflection error. Although the largest tracking error for both controllers is similar, the RMSE and average steady-state error under the robust controller are 3.66% and 36%, respectively, less than those of the PID. This proves that the designed robust controller outperforms the

Table 8.2: Controller comparison for step reference tracking.

<i>Approach</i>	RMSE (μm)	Average steady-state error (μm)
PID	6.28	1.11
Robust	6.05	0.62

PID controller in effectively and robustly reducing the steady-state error of the microactuator's actual deflection by considering the self-sensing modeling error. In practice, the actual deflection performance is of relevance, thus only the actual deflection is provided.

The advantages of the robust controller in terms of settling time over the PID controller can be noticed from Fig. 8.5(a), although there is higher overshoot with the robust controller. A closer examination of the controller efforts, quantified here as the amount of current change, reveals that the robust controller performs slightly higher work than the PID for longer time duration (see Fig. 8.6(b)), which explains the transient differences.

8.4.2 Multi-frequency Signals Reference Tracking

Experiments involving multisinusoidal reference inputs were carried out to study the performance of the microactuator under continuous input changes. For this experiment, the sum of three different sinusoidal waveforms with frequencies of 0.001, 0.005, and 0.01 Hz, maximum amplitude of $20 \mu\text{m}$ and an offset of $35 \mu\text{m}$ was chosen as the input signal. Fig. 8.7 shows the actual deflection of the microactuator as a function of time with PID and robust control. From the observed data, it is seen that the robust controller performance is better than that of the PID. This is more evident by looking at the tracking errors and control efforts under the two controllers, which are shown in Fig. 8.8(a) and (b), respectively.

The values for RMSE and SD calculated for this experiment are summarized in Table 8.3. It

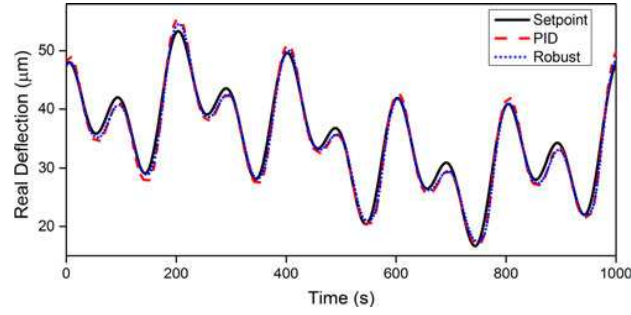


Figure 8.7: Microactuator deflection response to a multisinusoidal reference input under PID and robust control.

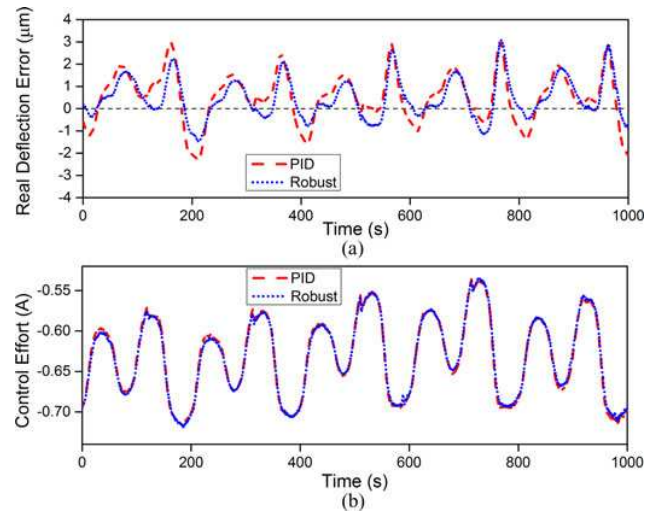


Figure 8.8: (a) Actual deflection error and (b) controller effort for the PID and robust control approaches in the multisinusoidal reference tracking experiment.

Table 8.3: Controller comparison for multisinusoidal reference tracking.

<i>Approach</i>	Largest error (μm)	RMSE (μm)	Control effort SD (mA)
PID	3.00	1.24	49.1
Robust	3.07	1.02	48.2

can be calculated that the robust controller has around 18% less tracking RMSE and 1.8% less control effort than the PID controller. The effectiveness of the robust controller in reducing the steady-state error of the actual deflection is again verified experimentally.

8.4.3 Noise Rejection for Multi-frequency Signals Reference Tracking

In order to study the robustness of the H_∞ controller to environmental disturbances, modeled in Fig. 8.3(b) as I_d , a white noise signal with maximum value of ± 0.01 A and band-limit of 8 Hz was added to the controller effort. The same input signal used in the multisinusoidal reference input experiment without noise was adopted for this study (see Fig. 8.7). The white noise amplitude corresponded to 25% of the total current change observed in Fig. 8.8(b), which represented an overestimate of reallife noise signals due to current variations. Fig.8.9(a) shows the actual deflections of the microactuator with the noisy input under robust and PID control. It is seen from the observed data that the closed-loop deflection system under the H_∞ controller performs robustly against noise disturbances better than with the PID. This difference in performance is evident in Fig. 8.9(b) where the largest error between actual deflection and reference input with robust control is $3.07 \mu\text{m}$ and for PID is $4.28 \mu\text{m}$. This translates to a 28% decrease in the largest error for the robust controller. While the largest tracking error for PID controller under noisy tracking is 43% larger than the noiseless tracking case, the robust controller ends with not evident deterioration in the largest tracking error. Fig. 8.10(a) shows the control effort applied by both controllers in these

Table 8.4: Controller comparison for multisinusoidal reference tracking with noise.

<i>Approach</i>	Largest error (μm)	RMSE (μm)	Control effort SD (mA)
PID	4.28	1.47	33.6
Robust	3.07	0.96	31.6

experiments and, for clarity, a separate plot in Fig.8.10(b) shows the Peltier input, which includes the controller effort and the white noise. Table 8.4 shows the RMSE and control effort SD values for this experiment. For the system with robust control, the RMSE is 34.7% less than with PID, and the control effort SD is 6% less. Although the robust control RMSE in this experiment is close to the one obtained without noise (see Table 8.3), an increase of 18.6% is observed for the system under PID control. This verifies the robust performance and stability of the controller to compensate for deflection control, not only for modeling errors, but also against environmental disturbances.

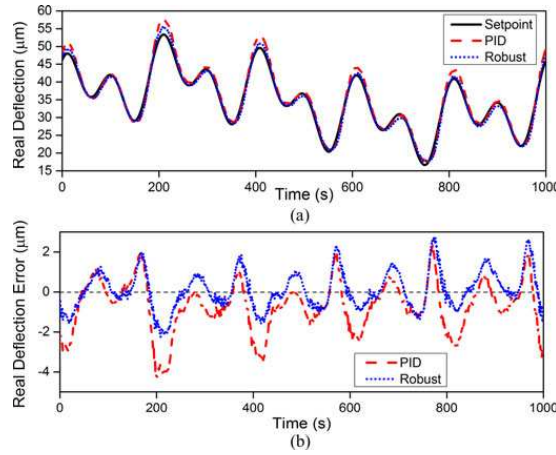


Figure 8.9: (a) Microactuator deflection response and (b) error under a multisinusoidal reference input under PID and robust control with induced white noise in current input to the system.

In addition to the RMSE and the maximum tracking errors, we have further conducted fast Fourier transform of the tracking errors under the PID and H_∞ controllers, for the scenarios with and without injected actuation noises. As shown in Fig. 8.11, the tracking error under the H_∞

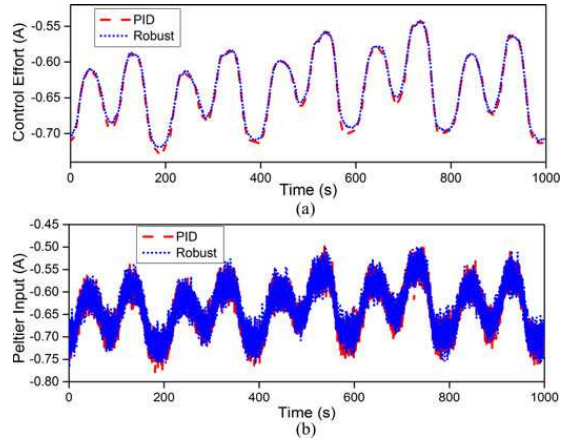


Figure 8.10: (a) Control effort and (b) Peltier input for a multisinusoidal reference input under PID and robust closed-control with induced white noise in current input to the system.

controller is lower than that under the PID controller for all frequency components. This is true for both the cases with and without noise.

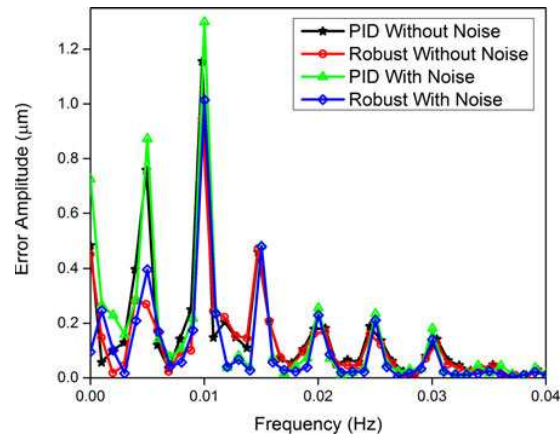


Figure 8.11: Frequency spectrum analysis of the tracking errors under the robust controller and the PID controller, for scenarios with and without injected actuation noise.

Chapter 9

Conclusions and Future Work

9.1 Conclusions

In this work, the modeling, identification, and control of hysteretic systems have been explored. A few new contributions have been made to the modeling and control of VO₂ microactuators.

First, tools from information theory are utilized to optimally compress the Preisach operator and the GPI model under a given complexity constraint. The compressed hysteresis models achieve high fidelity while maintaining relatively low calculation and storage complexity. While due to the particular setting of the Preisach plane, the optimal compression of the Preisach operator involves an exhaustive search, the optimal compression of the GPI model is reformulated as an optimal control problem and solved with dynamic programming. The proposed schemes are verified with simulation results as well as experimental results, where the hysteresis between VO₂ resistance and temperature is modeled.

Second, identification of the Preisach operator is studied under the compressive sensing framework that requires fewer measurements. The proposed approach adopts the DCT transform of the output data to obtain a sparse vector, where the order of all the output data is assumed to be known. The model parameters can be efficiently reconstructed using the proposed scheme. The least-squares scheme is also implemented as a comparison. The proposed identification approach is shown to have better performance than the least-squares scheme through both simulation and experiments involving a VO₂-integrated microactuator.

Third, a physics-motivated non-monotonic hysteresis model that accounts for the two competing actuation mechanisms is presented. The first mechanism is the stress resulting from structural changes in VO_2 , which is modeled with a monotonic Preisach operator or a GPI model. The second mechanism is the differential thermal expansion effect. Efficient inverse compensation schemes are developed for the proposed non-monotonic hysteresis models.

Fourth, self-sensing feedback control for VO_2 microactuator is studied. The proposed composite self-sensing approach exploits the physical understanding that both the resistance and the deflection have different hysteretic relationships with the temperature. A concept of temperature surrogate is exploited in the algorithm. The self-sensing scheme is validated experimentally in feedback control, where a proportional-integral controller is used.

Finally, an H_∞ robust controller is designed and implemented for precision deflection control, where the uncertainties produced by the hysteresis between the deflection and the temperature input and the error in the self-sensing model are accommodated. Here we take a simpler self-sensing approach that models the deflection as a polynomial function of the resistance. The proposed robust control approach is experimentally demonstrated to be able to mitigate the impact of the self-sensing error and other disturbances.

9.2 Future Work

First, for the compressive sensing-based identification of the Preisach operator, the transformation of the density will be further studied to generate sparser signals. The use of Moore-Penrose pseudoinverse [81] will be studied for cases where the input sequence has a different number of entries from the number of model parameters and where the matrix S is not invertable. Compressive sensing-based identification will also be explored for other hysteresis models, such as the GPI

model.

Second, we will examine the performance of the composite self-sensing scheme in advanced control of VO₂ microactuators. For example, robust control methods will be explored to minimize the impact of the self-sensing error on the tracking performance. In addition, the current composite self-sensing model is rate-independent. While it has demonstrated good performance overall in tracking control, the mild rate-dependence suggests that a rate-dependent (for example, accommodating the structural dynamics of the actuator), composite hysteresis model could offer further enhanced tracking performance at higher frequencies.

APPENDICES

Appendix A

Review of the Preisach Operator

For a more detailed treatment of Preisach operator, readers are referred to [9, 11, 12].

Preisach Operator

A Preisach operator consists of a weighted superposition of a continuum of basic hysteretic elements, called Preisach *hystérons*. A generic Preisach hysteron, $\gamma_{\beta,\alpha}$, is a delayed relay characterized by a pair of thresholds (β, α) . The output evolves with the input v , where an initial condition $\zeta_0(\beta, \alpha) \in \{-1, 1\}$ is needed to fully describe the behavior of $\gamma_{\beta,\alpha}$:

$$\gamma_{\beta,\alpha}[v(\cdot); \zeta_0(\beta, \alpha)] = \begin{cases} +1 & \text{if } v > \alpha \\ -1 & \text{if } v < \beta \\ \zeta_0(\beta, \alpha) & \text{if } \beta \leq v \leq \alpha \end{cases}, \quad (\text{A.1})$$

where $v(\cdot)$ denotes the input history $v(\tau)$, $0 \leq \tau \leq t$.

The output of a Preisach operator Γ , with input v and initial condition $\zeta_0 = \{\zeta_0(\beta, \alpha), \beta \leq \alpha\}$ can then be represented as:

$$u(t) = \Gamma[v; \zeta_0](t) = \int_{\mathcal{P}_0} \mu(\beta, \alpha) \gamma_{\beta,\alpha}[v; \zeta_0(\beta, \alpha)](t) d\beta d\alpha, \quad (\text{A.2})$$

where μ is a measurable density function typically assumed to be nonnegative. Each point (β, α) in

the *Preisach plane*, defined as $\mathcal{P} = \{(\beta, \alpha) : \beta \leq \alpha\}$, is identified with the hysteron $\gamma_{\beta, \alpha}$. Because of the input range constraints or physical saturation (e.g., complete phase transition beyond certain input range for VO₂), it often suffices to consider μ with finite support $\{(\beta, \alpha) : v_{min} \leq \beta \leq \alpha \leq v_{max}\}$ in $\mathcal{P}$. The state of the Preisach operator, namely, the outputs of all hysterons can be captured by *memory curve*, a staircase-structured line in $\mathcal{P}$ separating hysterons with output +1 from those with output -1.

Discretization of Preisach Operator

The density function μ of hysterons is the parameter of the Preisach operator. For parameter identification, a discretization step is typically involved to obtain a finite number of parameter values. For the Preisach operator, one scheme is to approximate the operator with a finite number of hysterons located at the center of uniformly spaced lattice cells in the Preisach plane [12]. This is equivalent to approximating the weighting function by a sum of impulse functions located at the cell centers, which results in a discontinuous output under a continuous input. An alternative scheme, still based on uniform discretization of the Preisach plane, approximates the density by a piecewise constant function – the density value is constant within each lattice cell but could vary from cell to cell [7]. Fig. A.1 shows an example of uniform discretization of Preisach plane. Under this scheme, the Preisach operator has $M(M+1)/2$ density parameters, where M is the level of uniform discretization along α (or equivalently, β) direction in the Preisach plane. This scheme produces a continuous output under a continuous input; furthermore, efficient schemes for the identification [25] and inversion [9] are available.

The output of the Preisach model (in the discrete-time setting) at time n can be expressed as:

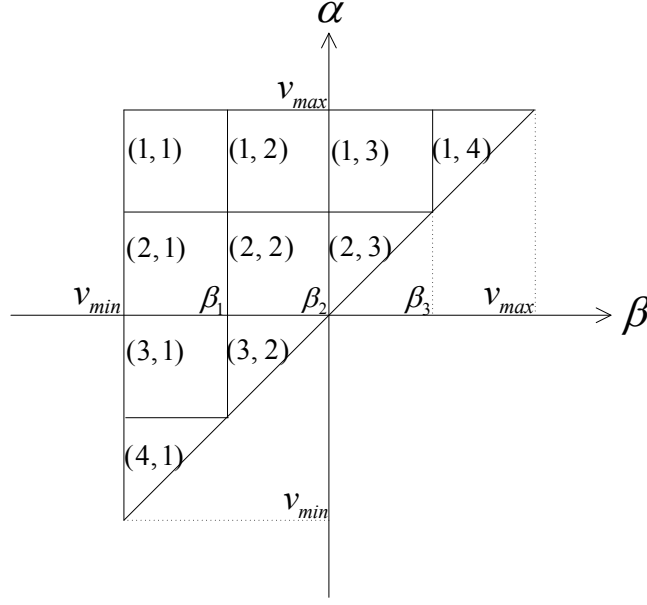


Figure A.1: Illustration of uniform discretization of the Preisach plane, where the discretization level $M = 4$.

$$\tilde{u}(n) = u_c + \sum_{i=1}^M \sum_{j=1}^{M+1-i} \mu_{ij} s_{ij}[n], \quad (\text{A.3})$$

where u_c represents a constant contribution from hysterons outside the active Preisach plane, μ_{ij} is the density value for cell (i, j) , and $s_{ij}[n]$ represents the *signed* area of the cell (i, j) , namely, its area occupied by hysterons with output $+1$ minus that occupied by hysterons with output -1 . The calculation complexity is $O(M^2)$, and when M is large, the calculation and storage cost can be prohibitive.

Inversion

A predominant class of control approaches involve approximate cancellation of the hysteresis effect through inversion [6, 9, 10]. By constructing an approximate (right) inverse to the Preisach operator (Fig. A.2), the hysteresis effect can be (mostly) cancelled.

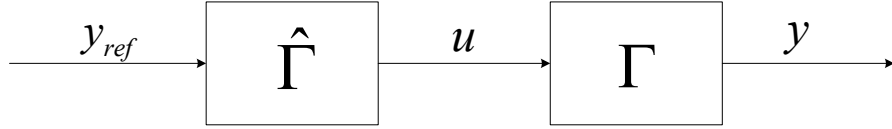


Figure A.2: Inverse compensation of hysteresis.

The inversion scheme used in [9] exploits the piecewise constant structure of the density function and the piecewise monotonicity property of the operator. While the scheme was initially developed for a Preisach operator with uniform discretization of the Preisach plane, it is easily modified to accommodate an operator with nonuniform discretization, without increasing computational complexity.

Appendix B

Review of the Prandtl-Ishlinskii Models

A brief overview of the classical Prandtl-Ishlinskii (CPI) model and generalized Prandtl-Ishlinskii (GPI) models are provided. Readers are referred to [13–15, 22] for more details.

Classical Prandtl-Ishlinskii (CPI) Model

The CPI model consists of a weighted superposition of basic play (or stop) operators. The CPI model is limited to modeling symmetric and non-saturated hysteresis. As illustrated in Fig. B.1(a), the play operator is characterized by its radius r . For a given input function $v(t)$, the output $w(t)$ of a play operator with radius r and initial condition $w(t^-)$ is defined as

$$w(t) = F_r[v](t) = f_r(v(t), F_r[v](t^-)), \quad (\text{B.1})$$

where

$$f_r(v(t), w(t^-)) = \begin{cases} \max(v(t) - r, w(t^-)), & \text{if } v(t) > v(t^-) \\ \min(v(t) + r, w(t^-)), & \text{if } v(t) < v(t^-) \\ w(t^-), & \text{if } v(t) = v(t^-) \end{cases}, \quad (\text{B.2})$$

and $t^- = \lim_{\varepsilon > 0, \varepsilon \rightarrow 0} t - \varepsilon$.

The output of a CPI model is expressed as an integral in the following form:

$$y_P(t) = \int_0^{R_P} p(r) F_r[v](t) dr, \quad (\text{B.3})$$

where $p(r)$ is the weighting function of the Prandtl-Ishlinskii model, which is usually chosen to be non-negative, and R_P represents the maximum play radius.

For practical implementation, the CPI model is represented as a weighted summation of a finite number of play operators as follows:

$$y_P(t) = p(r_0)v + \sum_{j=1}^N p(r_j) F_{r_j}[v](t), \quad (\text{B.4})$$

where $r_j > 0$ is the play radius of the j -th play operator, $p(r_j)$ is the corresponding weight, and N denotes the number of play operators.

Generalized Prandtl-Ishlinskii (GPI) Model

The GPI hysteresis model can capture complex hysteresis loops with both asymmetry and output saturation [13–15, 28].

Following a similar treatment as in [13, 28], a generalized play operator with radius r is defined by (see Fig. B.1(b))

$$w(t) = F_r^\gamma[v](t) = f_r^\gamma(v(t), F_r^\gamma[v](t^-)), \quad (\text{B.5})$$

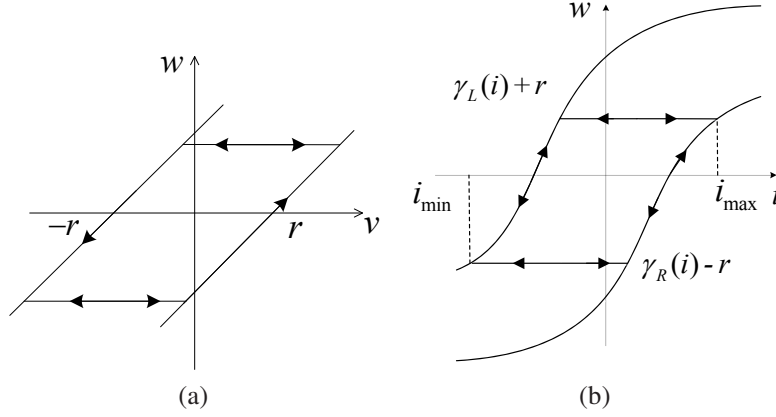


Figure B.1: Input-output relationships of (a) a classical play operator with radius r ; (b) a generalized play operator with radius r (shown as solid curves).

where $f_r^\gamma(t, w(t))$ is defined as

$$f_r^\gamma(v(t), w(t^-)) = \begin{cases} \max(\gamma_L(v(t)) - r, w(t^-)), & \text{if } v(t) > v(t^-) \\ \min(\gamma_R(v(t)) + r, w(t^-)), & \text{if } v(t) < v(t^-), \\ w(t^-), & \text{if } v(t) = v(t^-) \end{cases} \quad (\text{B.6})$$

where $\gamma_L(\cdot)$, and $\gamma_R(\cdot)$ are two envelope functions that are strictly increasing. The envelope functions describe the properties of the play operators. For any radius $r \geq 0$ and input $v(t)$, the condition $\gamma_L(v(t)) + r \geq \gamma_R(v(t)) - r$ needs to be satisfied in order to meet the order preservation property of hysteresis behavior [27].

The output of a GPI model can be expressed in the integral form as

$$y_P(t) = \int_0^{R_p} p(r) F_r^\gamma[v](t) dr. \quad (\text{B.7})$$

Similar to the CPI case, a discrete-version of the GPI model can be written as

$$y_{P_\gamma}(t) = \sum_{j=0}^N p(r_j) F_{r_j}^\gamma[v](t). \quad (\text{B.8})$$

When $\gamma_L(v(t)) = \gamma_R(v(t))$, the GPI model can be utilized to model symmetric hysteresis; when $D(\cdot)$ is linear and $\gamma_L(v(t)) = \gamma_R(v(t)) = v(t)$, the GPI model degenerates to a CPI model.

Inversion

Denote Ψ as the GPI model, which can be written as

$$y_d = \Psi[v](t) = \sum_{j=0}^N p(r_j) F_r^\gamma[v](t), \quad (\text{B.9})$$

where y_d is the desired output of the GPI model, and denote $\hat{\Psi}^{-1}$ as its approximate inverse. Then ideally,

$$y = \Psi \circ \hat{\Psi}^{-1}[y_d](t) \approx y_d, \quad (\text{B.10})$$

is satisfied, where y is the actual output of the GPI model Ψ , and y_d is the desired output of the generalized model. Note that in inverse compensation, y_d is used as the input for the inverse model $\hat{\Psi}^{-1}$. “ $\circ$ ” denotes the composition of functions or operators. One can write

$$y(t) = \Psi[v](t) = \begin{cases} \Psi[v](t) = \Pi \circ \gamma_R(v(t)), & \text{if } v(t) > v(t^-) \\ \Psi[v](t) = \Pi \circ \gamma_L(v(t)), & \text{if } v(t) < v(t^-). \\ y(t^-) & \text{if } v(t) = v(t^-) \end{cases} \quad (\text{B.11})$$

where Π denotes the classical PI model.

Due to the invertibility of the envelope functions γ_L and γ_R , Eq. (B.11) can be expressed as

$$v(t) = \begin{cases} \gamma_R^{-1} \circ \Pi^{-1} \circ y(t), & \text{if } y(t) > y(t^-) \\ \gamma_L^{-1} \circ \Pi^{-1} \circ y(t), & \text{if } y(t) < y(t^-). \\ v(t^-), & \text{if } y(t) = y(t^-) \end{cases} \quad (\text{B.12})$$

The inverse of the GPI model is written as [28]

$$\Psi^{-1}[y_d](t) = \begin{cases} \gamma_R^{-1} \circ \Pi^{-1}[y_d](t), & \text{if } y_d(t) > y_d(t^-) \\ \gamma_L^{-1} \circ \Pi^{-1}[y_d](t), & \text{if } y_d(t) < y_d(t^-), \\ \Psi^{-1}[y_d](t^-) & \text{if } y_d(t) = y_d(t^-) \end{cases} \quad (\text{B.13})$$

where Π^{-1} is the inversion of the CPI model, the expression of which can be found in [20].

The inverse of the CPI model Π with form of Eq. (B.4) is another CPI operator with different parameters:

$$\Pi^{-1}[y](t) = \hat{p}(r_0)y(t) + \sum_{i=1}^N \hat{p}(\hat{r}_i)F_{\hat{r}_i}[y](t), \quad (\text{B.14})$$

where

$$\hat{r}_j = p(r_0)r_j + \sum_{i=1}^{j-1} p(r_i)(r_j - r_i), j \geq 1, \quad (\text{B.15})$$

$$\hat{p}(r_0) = \frac{1}{p(r_0)}, \quad (\text{B.16})$$

and

$$\hat{p}(\hat{r}_i) = -\frac{p(r_i)}{(p(r_0) + \sum_{j=1}^i p(r_j))(p(r_0) + \sum_{j=1}^{i-1} p(r_j))}, \quad (\text{B.17})$$

for $i = 1, \dots, N$.

BIBLIOGRAPHY

BIBLIOGRAPHY

- [1] J. A. Ewing, “On the production of transient electric currents in iron and steel conductors by twisting them when magnetized or by magnetising them when twisted,” *Proc. Royal Soc. Lond.*, vol. 33, pp. 21–23, 1881.
- [2] J. Das, M. Ho, J. Zikherman, C. Govern, M. Yang, A. Weiss, A. K. Chakraborty, and J. P. Roose, “Digital signaling and hysteresis characterize ras activation in lymphoid cells,” *Cell*, vol. 136, no. 2, pp. 337–351, 2009.
- [3] M. Grinfeld, R. Cross, and H. Lamba, “Hysteresis and economics - Taking the economic past into account,” *IEEE Control Syst. Mag.*, vol. 29, no. 1, pp. 30–43, 2009.
- [4] D. C. Jiles and D. L. Atherton, “Theory of ferromagnetic hysteresis,” *J. Magn. Magn. Mater.*, vol. 61, pp. 48–60, 1986.
- [5] P. Ge and M. Jouaneht, “Generalized Preisach model for hysteresis nonlinearity of piezoceramic actuators,” *Precis. Eng.*, vol. 20, no. 2, pp. 99–111, 1997.
- [6] Z. Chen, X. Tan, and M. Shahinpoor, “Quasi-static positioning of ionic polymer-metal composite (IPMC) actuators,” in *Proc. of the IEEE/ASME I. Conf. on Adv. Intel. Mech.*, 2005.
- [7] J. Ahrens, X. Tan, and H. K. Khalil, “Multirate sampled-data output feedback control with application to smart material actuated systems,” *IEEE Trans. Automat. Contr.*, vol. 54, no. 11, pp. 2518–2529, 2009.
- [8] E. Merced, R. Cabrera, N. Davila, F. E. Fernández, and N. Sepúlveda, “A micro-mechanical resonator with programmable frequency capability,” *Smart Mater. Struct.*, vol. 21, no. 3, 2012.
- [9] X. Tan and J. Baras, “Modeling and control of hysteresis in magnetostrictive actuators,” *Automatica*, vol. 40, no. 9, pp. 1469–1480, 2004.
- [10] G. Song, J. Zhao, X. Zhou, and J. de Abreu-Garcia, “Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model,” *IEEE/ASME Trans. Mech.*, vol. 10, no. 2, pp. 198–209, 2005.

- [11] I. Mayergoyz and G. Friedman, “Generalized Preisach model of hysteresis,” *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 212–217, 1988.
- [12] X. Tan and J. Baras, “Adaptive identification and control of hysteresis in smart materials,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 6, pp. 827–839, 2005.
- [13] M. Al Janaideh, S. Rakheja, and C.-Y. Su, “A generalized Prandtl-Ishlinskii model for characterizing the hysteresis and saturation nonlinearities of smart actuators,” *Smart Mater. Struct.*, vol. 18, no. 4, 2009.
- [14] A. Visintin, *Differential models of hysteresis*. Springer, 1994.
- [15] M. Brokate and J. Sprekels, *Hysteresis and phase transitions*. Springer, 1996.
- [16] Q. Wang and C. Su, “Robust adaptive control of a class of nonlinear systems including actuator hysteresis with Prandtl-Ishlinskii presentations,” *Automatica*, vol. 42, no. 5, pp. 859–867, 2006.
- [17] J. Oh and D. Bernstein, “Semilinear Duhem model for rate-independent and rate-dependent hysteresis,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 5, pp. 631–645, 2005.
- [18] F. Ikhouane and J. Rodellar, “A linear controller for hysteretic systems,” *IEEE Trans. Automat. Contr.*, vol. 51, no. 2, pp. 340–344, 2006.
- [19] T. Vo-Minh, T. Tjahjowidodo, H. Ramon, and H. V Brussel, “A new approach to modeling hysteresis in a pneumatic artificial muscle using the Maxwell-slip model,” *IEEE/ASME Trans. Mech.*, vol. 16, no. 1, pp. 177–186, 2011.
- [20] P. Krejci and K. Kuhnen, “Inverse control of systems with hysteresis and creep,” *IEE P-Contr. Theor. Ap.*, vol. 148, no. 3, pp. 185–192, 2001.
- [21] A. Esbrook, X. Tan, and H. Khalil, “Control of systems with hysteresis via servocompensation and its application to nanopositioning,” *IEEE Trans. Contr. Syst. T.*, vol. 21, no. 3, pp. 725–738, 2013.
- [22] K. Kuhnen, “Modeling, identification and compensation of complex hysteretic nonlinearities: A modified Prandtl-Ishlinskii approach,” *Eur. J. Control*, vol. 9, no. 4, pp. 407–418, 2003.
- [23] M. Ruderman, “Direct recursive identification of the preisach hysteresis density function,” *J. Magn. Magn. Mater.*, vol. 348, pp. 22–26, 2013.

- [24] Z. Li, C.-Y. Su, and T. Chai, "Compensation of hysteresis nonlinearity in magnetostrictive actuators with inverse multiplicative structure for Preisach model," *IEEE Trans. Autom. Sci. Eng.*, vol. 11, no. 2, pp. 613–619, 2014.
- [25] X. Tan, R. Venkataraman, and P. S. Krishnaprasad, "Control of hysteresis: Theory and experimental results," in *Proc. of the SPIE, Smart Structures and Materials : Modeling, Signal Processing, and Control in Smart Structures*, 2001.
- [26] J. Zhang, E. Merced, N. Sepúlveda, and X. Tan, "Modeling and inverse compensation of non-monotonic hysteresis in VO₂-coated microactuators," *IEEE/ASME Trans. Mech.*, vol. 19, no. 2, pp. 579–588, 2014.
- [27] R. Iyer and X. Tan, "Control of hysteretic systems through inverse compensation: Inversion algorithms, adaptation, and embedded implementation," *IEEE Contr. Syst. Mag.*, vol. 29, no. 1, pp. 83–99, 2009.
- [28] M. Al Janaideh, R. Subhash, and C. Y. Su, "An analytical generalized Prandtl-Ishlinskii model inversion for hysteresis compensation in micropositioning control," *IEEE/ASME Trans. Mech.*, vol. 16, no. 4, pp. 734–744, 2011.
- [29] J. Ouyang and Y. Zhu, "Z-shaped MEMS thermal actuators: Piezoresistive self-sensing and preliminary results for feedback control," *J. Microelectromech. S.*, vol. 21, no. 3, pp. 596–604, 2012.
- [30] C.-C. Lan, C.-M. Lin, and C.-H. Fan, "A self-sensing microgripper module with wide handling ranges," *IEEE/ASME Trans. Mech.*, vol. 16, no. 1, pp. 141–150, 2011.
- [31] M. Rakotondrabe, I. Ivan, S. Khadraoui, C. Clevy, P. Lutz, and N. Chaillet, "Dynamic displacement self-sensing and robust control of cantilever piezoelectric actuators dedicated for microassembly," in *Advanced Intelligent Mechatronics (AIM), 2010 IEEE/ASME International Conference on*, 2010, pp. 557–562.
- [32] T. Takigami, K. Oshima, and Y. Hayakawa, "Application of self-sensing actuator to control of a cantilever beam," in *American Control Conference, 1997. Proceedings of the 1997*, vol. 3, 1997, pp. 1867–1872.
- [33] W. Zhang, J. Qiu, and J. Tani, "Robust vibration control of a plate using self-sensing actuators of piezoelectric patches," *Journal of Intelligent Material Systems and Structures*, vol. 15, no. 12, pp. 923–931, 2004.
- [34] U. Boettcher, L. Matthes, B. Knigge, R. A. de Callafon, and F. E. Talke, "Suppression of cross-track vibrations using a self-sensing micro-actuator in hard disk drives," *Microsys-*

tem Technologies-Micro- and Nanosystems Information Storage and Processing Systems, vol. 18, no. 9-10, pp. 1309–1317, 2012.

- [35] M. Marezio, D. B. McWhan, J. P. Remeika, and P. D. Dernier, “Structural aspects of the metal-insulator transitions in Cr-doped VO₂,” *Phys. Rev. B*, vol. 5, pp. 2541–2551, 1972.
- [36] A. Rua, F. E. Fernández, and N. Sepúlveda, “Bending in VO₂-coated microcantilevers suitable for thermally activated actuators,” *J. Appl. Phys.*, vol. 107, no. 7, 2010.
- [37] A. Rua, F. Fernández, and N. Sepúlveda, “Opto-electronic and full-optical multiple state memory response in phase-change materials,” *J. Appl. Phys.*, vol. 108, no. 11, 2010.
- [38] E. Merced, R. Cabrera, H. Coy, F. E. Fernández, and N. Sepúlveda, “Frequency tuning of VO₂-coated buckled microbridges,” *J. Microelectromech. Syst.*, vol. 20, no. 3, pp. 558–560, 2011.
- [39] R. Cabrera, E. Merced, N. Sepúlveda, and F. E. Fernández, “Dynamics of photothermally driven VO₂-coated microcantilevers,” *J. Appl. Phys.*, vol. 110, no. 9, 2011.
- [40] E. Merced, N. Dávila, D. Torres, R. Cabrera, F. E. Fernández, and N. Sepúlveda, “Photothermal actuation of VO₂:Cr-coated micro-cantilevers in air and aqueous media,” *Smart Mater. and Struct.*, vol. 21, 105009, 2012.
- [41] R. Cabrera, E. Merced, N. Dávila, F. E. Fernández, and N. Sepúlveda, “A multiple-state micro-mechanical programmable memory,” *Microelectron. Eng.*, vol. 88, no. 11, pp. 3231–3234, 2011.
- [42] S. Bashash and N. Jalili, “Robust adaptive control of coupled parallel piezo-flexural nanopositioning stages,” *IEEE-ASME Tran. Mech.*, vol. 14, no. 1, pp. 11–20, 2009.
- [43] Y. L. Zhang, M. L. Han, M. Y. Yu, C. Y. Shee, and W. T. Ang, “Automatic hysteresis modeling of piezoelectric micromanipulator in vision-guided micromanipulation systems,” *IEEE-ASME Tran. Mech.*, vol. 17, no. 3, pp. 547–553, 2012.
- [44] Y. Qin, Y. Tian, and D. Zhang, “A novel direct inverse modeling approach for hysteresis compensation of piezoelectric actuator in feedforward applications,” *IEEE-ASME Tran. Mech.*, vol. 18, no. 3, pp. 981–989, 2013.
- [45] Y. Cao, C. X B, and J. Y. Peng, “An inversion-based model predictive control with an integral-of-error state variable for piezoelectric actuators,” *IEEE-ASME Tran. Mech.*, vol. 18, no. 3, pp. 895–904, 2013.

- [46] K. K. Leang, Q. Zou, and S. Devasia, "Feedforward control of piezoactuators in atomic force microscope systems," *IEEE Contr. Syst. Mag.*, vol. 29, no. 1, pp. 70–82, 2009.
- [47] R. B. Gorbet, K. A. Morris, and D. Wang, "Passivity-based stability and control of hysteresis in smart actuators," *IEEE Tran. Contr. Syst. T.*, vol. 9, no. 1, pp. 5–16, 2001.
- [48] B. Drincic, X. Tan, and D. S. Bernstein, "Why are some hysteresis loops shaped like a butterfly?" *Automatica*, vol. 47, no. 12, pp. 2658–2664, 2011.
- [49] G. Y. Gu, L. M. Zhu, and C. Y. Su, "Modeling and compensation of asymmetric hysteresis nonlinearity for piezoceramic actuators with a modified Prandtl-Ishlinskii model," *IEEE Trans. Ind. Electron.*, vol. 61, no. 3, pp. 1583–1595, 2014.
- [50] K. Suzuki, H. Funaki, and Y. Naruse, "MEMS optical microphone with electrostatically controlled grating diaphragm," *Meas. Sci. Technol.*, vol. 17, no. 4, pp. 819–824, 2006.
- [51] T. McPherson and J. Ueda, "A force and displacement self-sensing piezoelectric MRI-compatible tweezer end effector with an on-site calibration procedure," *IEEE/ASME Trans. Mech.*, vol. 19, no. 2, pp. 755–764, 2014.
- [52] I. A. Ivan, M. Rakotondrabe, P. Lutz, and N. Chaillet, "Current integration force and displacement self-sensing method for cantilevered piezoelectric actuators," *Rev. Sci. Instrum.*, vol. 80, no. 12, p. 126103, 2009.
- [53] C.-C. Lan and C.-H. Fan, "An accurate self-sensing method for the control of shape memory alloy actuated flexures," *Sensor Actuat. A-Phys.*, vol. 163, no. 1, pp. 323–332, 2010.
- [54] S. Chaitanya and K. Dhanalakshmi, "Demonstration of self-sensing in shape memory alloy actuated gripper," in *Proceedings of IEEE International Symposium on Intelligent Control (ISIC)*, 2013, pp. 218–222.
- [55] M. Ferdous, M. Rashid, M. Bhuiyan, A. Muthalif, and M. Hasan, "Novel design of a self powered and self sensing magneto-rheological damper," *Smart Mater. Struct.*, vol. 53, no. 1, p. 012048, 2013.
- [56] E. Merced, D. Torres, X. Tan, and N. Sepulveda, "An electrothermally actuated VO₂-based MEMS using self-sensing feedback control," *J. Microelectromech. S.*, vol. 24, no. 1, pp. 100–107, 2015.
- [57] S. Kullback and R. Leibler, "On information and sufficiency," *Ann. Math. Stat.*, vol. 22, no. 1, pp. 79–86, 1951.

- [58] X. Nguyen, M. Wainwright, and M. Jordan, “Estimating divergence functionals and the likelihood ratio by convex risk minimization,” *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5847–5861, 2010.
- [59] R. Shankar, H. Mobahi, A. Yang, S. Sastry, and Y. Ma, “Natural image segmentation with adaptive texture and boundary encoding,” in *Proceedings of Asian Conference on Computer Vision (ACCV)*, 2009.
- [60] Y. Wang, K. Liu, Q. Hao, X. Wang, D. Lau, and L. Hassebrook, “Robust active stereo vision using Kullback-Leibler divergence,” *IEEE Trans. Pattern Anal.*, vol. 34, no. 3, pp. 548–563, 2012.
- [61] D. Goldberg, *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley, 1989.
- [62] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, “Optimization by simulated annealing,” *Science*, vol. 220, no. 4598, pp. 671–680, 1983.
- [63] D. Bertsekas, *Dynamic Programming and Optimal Control: Volume I*. Athena Scientific, 2005, no. v.1.
- [64] C. Shannon, “A mathematical theory of communication,” *AT&T Tech. J.*, vol. 27, pp. 379–423, 623–656, 1948.
- [65] K. J. Balakrishnan and N. A. Touba, “Relationship between entropy and test data compression,” *IEEE Trans. Comput. Aid D.*, vol. 26, no. 2, pp. 386–395, 2007.
- [66] B. F. Wu and H. H. Hsu, “Entropy-constrained scalar quantization and minimum entropy with error bound by discrete wavelet transforms in image compression,” *IEEE Trans. Signal Proces.*, vol. 48, no. 4, pp. 1133–1143, 2000.
- [67] J. Zhang, E. Merced, N. Sepúlveda, and X. Tan, “Optimal compression of a generalized Prandtl-Ishlinskii operator in hysteresis modeling,” in *Proceedings of ASME Dynamic Systems and Control Conference*, Paper DSCC2013-3969, 2013.
- [68] E. Candes, J. Romberg, and T. Tao, “Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans. Inform. Theor.*, vol. 52, no. 2, pp. 489–509, 2006.
- [69] M. Rudelson and R. Vershynin, “On sparse reconstruction from Fourier and Gaussian measurements,” *Commun. Pur. Appl. Math.*, vol. 61, no. 8, pp. 1025–1045, 2008.

- [70] E. Candes and M. Wakin, “An introduction to compressive sampling,” *IEEE Signal Proc. Mag.*, vol. 25, no. 2, pp. 21–30, 2008.
- [71] E. J. Cands, J. K. Romberg, and T. Tao, “Stable signal recovery from incomplete and inaccurate measurements,” *Commun. Pur. Appl. Math.*, vol. 59, no. 8, pp. 1207–1223, 2006.
- [72] J. Zhao, B. Song, N. Xi, K. W. C. Lai, H. Chen, and C. Qu, “Compressive feedback based non-vector space control,” in *Proceedings of American Control Conference (ACC)*, 2012, pp. 4090–4095.
- [73] R. G. Moreno-Alvarado and M. Martinez-Garcia, “DCT-compressive sampling of frequency-sparse audio signals,” in *Proceedings of the World Congress on Engineering*, 2011, pp. 627–631.
- [74] C. Deng, W. Lin, B.-S. Lee, and C. Lau, “Robust image compression based on compressive sensing,” in *Proceedings of the IEEE International Conference on Multimedia and Expo (ICME)*, 2010, pp. 462–467.
- [75] B. Maxwell and S. Andersson, “A compressed sensing measurement matrix for atomic force microscopy,” in *Proceedings of American Control Conference (ACC)*, 2014, pp. 1631–1636.
- [76] M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, “Single-pixel imaging via compressive sampling,” *IEEE Signal Proc. Mag.*, vol. 25, no. 2, pp. 83–91, 2008.
- [77] Y. H. Chang and C. Tomlin, “Data-driven graph reconstruction using compressive sensing,” in *Proceedings of Decision and Control Conference (CDC)*, 2012, pp. 1035–1040.
- [78] S. M. OConnor, J. P. Lynch, and A. C. Gilbert, “Compressed sensing embedded in an operational wireless sensor network to achieve energy efficiency in long-term monitoring applications,” *Smart Mater. Struct.*, vol. 23, no. 8, p. 085014, 2014.
- [79] J. Wright, A. Yang, A. Ganesh, S. Sastry, and Y. Ma, “Robust face recognition via sparse representation,” *IEEE Trans. Pattern Anal.*, vol. 31, no. 2, pp. 210–227, 2009.
- [80] R. Toth, B. Sanandaji, K. Poolla, and T. Vincent, “Compressive system identification in the linear time-invariant framework,” in *Proceedings of Decision and Control and European Control Conference (CDC-ECC)*, 2011, pp. 783–790.
- [81] A. Ben-Israel and T. Greville, *Generalized Inverses: Theory and Applications*, ser. CMS Books in Mathematics. Springer New York, 2006.

- [82] J. Zhang, E. Merced, N. Sepúlveda, and X. Tan, “Optimal compression of generalized Prandtl-Ishlinskii hysteresis models,” *Automatica*, vol. 57, pp. 170–179, 2015.
- [83] D. Kucharczyk and T. Niklewski, “Accurate X-ray determination of the lattice parameters and the thermal expansion coefficients of VO₂ near the transition temperature,” *J. of Appl. Cryst.*, vol. 12, no. 4, pp. 370–373, 1979.
- [84] Y. Okada and Y. Tokumaru, “Precise determination of lattice-parameter and thermal-expansion coefficient of silicon between 300K and 1500K,” *J. Appl. Phys.*, vol. 56, no. 2, pp. 314–320, 1984.
- [85] I. Mayergoyz, *Mathematical models of hysteresis*. Springer-Verlag, 1991.
- [86] S. Timoshenko, “Analysis of bi-metal thermostats,” *J. Opt. Soc. Am. A.*, vol. 11, no. 3, pp. 233–255, 1925.
- [87] J. Nag, R. F. Haglund, Jr., E. A. Payzant, and K. L. More, “Non-congruence of thermally driven structural and electronic transitions in VO₂,” *J. Appl. Phys.*, vol. 112, no. 10, 2012.
- [88] D. Croft, G. Shed, and S. Devasia, “Creep, hysteresis, and vibration compensation for piezoactuators: Atomic force microscopy application,” *J. Dyn. Syst. Tran. ASME*, vol. 123, no. 1, pp. 35–43, 2001.
- [89] S. Devasia, E. Eleftheriou, and S. O. R. Moheimani, “A survey of control issues in nanopositioning,” *IEEE Tran. Contr. Syst. T.*, vol. 15, no. 5, pp. 802–823, 2007.
- [90] Y. Wu and Q. Zou, “Iterative control approach to compensate for both the hysteresis and the dynamics effects of piezo actuators,” *IEEE Tran. Contr. Syst. T.*, vol. 15, no. 5, pp. 936–944, 2007.
- [91] V. Istratescu, *Fixed Point Theory: An Introduction*, ser. Mathematics and Its Applications Series. Springer London, 2002, Chapter 7.
- [92] R. Cabrera, E. Merced, and N. Sepúlveda, “A micro-electro-mechanical memory based on the structural phase transition of VO₂,” *Phys. Status Solidi A*, vol. 210, no. 9, pp. 1704–1711, 2013.
- [93] Y. Zhou, X. Chen, C. Ko, Z. Yang, C. Mouli, and S. Ramanathan, “Voltage-triggered ultrafast phase transition in vanadium dioxide switches,” *IEEE Electron Device Lett.*, vol. 34, no. 2, pp. 220–222, 2013.

- [94] D. Leo, *Engineering Analysis of Smart Material Systems*, ser. Wiley InterScience. Wiley, 2007.
- [95] J. Zhang, E. Merced, N. Sepúlveda, and X. Tan, “Modeling and inverse compensation of hysteresis in vanadium dioxide using an extended generalized Prandtl-Ishlinskii model,” *Smart Mater. Struct.*, vol. 23, no. 12, p. 125017, 2014.
- [96] J. L. Arlett, J. R. Maloney, B. Gudlewski, M. Muluneh, , and M. L. Roukes, “Self-sensing micro and nanocantilevers with attonewton-scale force resolution,” *Nano Letters*, vol. 6, no. 5, pp. 1000–1006, 2006.
- [97] K. Zhou and J. Doyle, *Essentials of Robust Control*. Prentice Hall, 1998.
- [98] Q. Zhong, *Robust Control of Time-delay Systems*. Springer, 2006.
- [99] M. Rakotondrabe, Y. Haddab, and P. Lutz, “Quadrilateral modelling and robust control of a nonlinear piezoelectric cantilever,” *IEEE Trans. Contr. Syst. T.*, vol. 17, no. 3, pp. 528–539, 2009.
- [100] Z. Chen and X. Tan, “A control-oriented and physics-based model for ionic polymer–metal composite actuators,” *IEEE/ASME Trans. Mech.*, vol. 13, no. 5, pp. 519–529, 2008.
- [101] A. Packard and J. Doyle, “The complex structured singular value,” *Automatica*, vol. 29, no. 1, pp. 71 – 109, 1993.