# ROBOTIC FISH: DEVELOPMENT, MODELING, AND APPLICATION TO MOBILE SENSING

By

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#### ABSTRACT

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#### By

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Robotic fish are underwater robots that emulate locomotion of live fish through actuated fin and/or body movements. They are of increasing interest due to their potential applications such as aquatic environmental monitoring and robot-animal interactions.

In this work, several bio-inspired robotic fish prototypes have been developed that make use of periodic tail motions. A dynamic model for a tail-actuated robotic fish is presented by merging rigid-body dynamics with Lighthill's large-amplitude elongated-body theory. The model is validated with extensive experiments conducted on a robotic fish prototype. The role of incorporating the body motion in evaluating the tail-generated hydrodynamic forces is assessed, which shows that ignoring the body motion (as often done in the literature) results in significant overestimate of the thrust force and robot speed. By exploiting the strong correlation between the angle of attack and the tail-beat bias, a computationally efficient approach is further proposed to adapt the drag coefficients of the robotic fish.

It has been recognized that the flexibility of the body and fin structures has a pronounced impact on the swimming performance of biological and robotic fish. To analyze and utilize this trait, a novel dynamic model is developed for a robotic fish propelled by a flexible tail actuated at the base. The tail is modeled with multiple rigid segments connected in series through rotational springs and dampers. For comparison, a model using linear beam theory is created to capture the beam dynamics. Experimental result show that the two models have almost identical predictions when the tail undergoes small deformation, but only the proposed multi-segment model matches

the experimental measurement closely for all tail motions.

Motivated by the need for system analysis and efficient control of robotic fish, averaging of robots' dynamics is of interest. For dynamic models of robotic fish, however, classical or geometric averaging typically cannot produce an average model that is accurate and the in the meantime amenable to analysis or control design. In this work, a novel averaging approach for tail-actuated robotic fish dynamics is proposed. The approach consists of scaling the force and moment terms and then conducting classical averaging. Numerical investigation reveals that the scaling function for the force terms is a constant independent of tail-beat patterns, while the scaling function for the moment term depends linearly on the tail-beat bias. Existence and local stability of the equilibria for the average model are further analyzed. Finally, as an illustration of the utility of the average model, a semi-analytical framework is presented for obtaining steady turning parameters.

Sampling and reconstruction of a physical field using mobile sensor networks have recently received significant interest. In this work, an adaptive sampling framework is proposed to reconstruct aquatic environmental fields (e.g., temperature, or biomass of harmful algal blooms) using schools of robotic sensor platforms. In particular, it is assumed that the field of interest can be approximated by a low rank matrix, which is exploited for successive expansion of sampling area and analytical reconstruction of the field. For comparison, an Augmented Lagrange Multiplier optimization approach is also taken to complete the matrix reconstruction using a limited number of samples. Simulation results show that the proposed approach is more computationally efficient and requires shorter travel distances for the robots.

To Mom and Dad for their love and encouragement, and for building the foundation that supported me throughout this journey.

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# **Chapter 1**

# Introduction

Biomimetic systems have been receiving increasing attention from the robotics community, since natural organisms can provide important insights into the theory and design of engineer systems. For example, in the area of aquatic robots, the maneuverability and efficiency of live fish [3, 4] has motivated significant scientific interest over the past two decades in developing, modeling and controlling robotic fish [5–19]. In addition to providing platforms for underwater applications such as environmental monitoring [1, 20–23], these robots offer a means to study the behavior of live fish [24].

In this chapter, a review of robotic fish related state of arts is given in Section 1.1. The general organization of this report and major contributions of this work are clarified in Section 1.2 and 1.3, respectively. The author's publications during his Ph.D research are listed at Section 1.4

## **1.1 Robotic Fish as Mobile Sensing Platforms**

With five hundred million years of evolution, fish and other aquatic animals are endowed with a variety of morphological and structural features that enable them to move through water with speed, efficiency, and agility [3, 4]. The remarkable feats in biological swimming have stimulated extensive theoretical [18, 25, 26], experimental [27–31], and computational [32, 33] research by biologists, mathematicians, and engineers, in an effort to understand and mimic locomotion, maneuvering, and sensing mechanisms adopted by aquatic animals.

Over the past two decades, there has also been significant interest in developing underwater robots that propel and maneuver themselves like real fish do [5, 7, 11, 12, 19, 24, 34–49]. Often termed *robotic fish*, these robots provide an experimental platform for studying fish swimming, and hold strong promise for a number of underwater applications such as environmental monitoring [1]. Instead of using propellers, robotic fish accomplish swimming by deforming the body and/or fin-like appendages, mostly functioning as caudal fins and sometimes as pectoral fins. Body deformation and fin movements are typically achieved with motors. On the other hand, advances in smart materials have been explored to realize noiseless and compact actuation of robotic fish [6, 9, 12, 45, 50–55]. Fig. 1.1a shows a robotic fish swimming in an inland lake. Fig. 1.1b shows the close-up of another robot prototype, equipped with a dissolved oxygen (DO) sensor, global position system (GPS), internal measurement unit (IMU), and other electronic components, which is designed to monitor the DO level in aquafarms.

Aquatic environments have been facing various threats, and monitoring them and reconstructing aquatic process are of great importance to public health and biological balance. Manual sampling is still a common practice in environment monitoring, which is labor-intensive with difficulty in capturing dynamic phenomena of interest. Deployment of in-situ sensing with fixed or buoyed sensors or vertical profilers has been proposed [56–61]. However, since buoyed sensors have little freedom to move around, it cannot provide high spatial resolution to effectively adapt to the destruction of parts of the network, and will require a large number of units for capturing the distributed information of dynamic phenomena. The past decade has also seen great progress in the use of robotic technology, like autonomous underwater vehicles (AUVs) [62–68], and underwater gliders [69–71] for aquatic environmental sensing [63, 66–68, 72–84]. However, it is difficult to deploy many of them due to the high manufacturing and operational costs.

On the other hand, schools of autonomous robotic fish sensing platforms provide a competitive



(a)



Figure 1.1: Prototypes of robotic fish developed by Smart Microsystems Lab at Michigan State University [1]: (a) testing in a inland lake; (b) prototype for dynamically monitoring the dissolved oxygen level in aquafarms

alternative for the task of environment monitoring, with their low-cost, high-volume traits. However, the minimal human intervention and limited onboard resources prohibit the robots' ability to operate robustly in the unfriendly and unpredictable environment. This poses challenges across a wide spectrum, ranging from locomotion and maneuvering mechanisms, energy-efficient deigns, communication schemes, control and coordination, to sampling and reconstruction strategies.

This dissertation spans robotic fish development, dynamic modeling, averaging, nonlinear controller design, and an adaptive sampling framework to guide a group of robotic fish.

## 1.2 Organization

This dissertation is organized as follows. In Chapter 2, guidelines for designing robotic fish are provided and a detailed description of one robotic fish system is explained as a representative example. Then some other prototypes that the author made significant contributions to, e.g., a gliding robotic fish and a robotic predator are briefly discussed. The applications of these robots are also covered in this chapter. In Chapter 3, a complete dynamic model is presented for robotic fish actuated by a rigid caudal fin, where Lighthill's elongated-body theory is used to evaluate the tail-generated hydrodynamic forces. It shows that, the drag force and moment coefficients are strongly dependent on the tail-beat bias, and subsequently a novel scheme is proposed to adapt these coefficients and lift force based on the tail movement pattern. In Chapter 4, a model for robotic fish with a base-actuated flexible tail is developed. The tail is modeled as multiple rigid segments connected by springs and dampers, which is compared with the model based on the linear beam theory. In Chapter 5, a control-oriented averaging approach for tail-actuated robotic fish dynamics is developed, which is applied to a hybrid controller for target control. In Chapter 6, an adaptive sampling framework is developed for a group of robotic fish, where it is assumed that the field can be represented by a low-rank matrix. Motivated by motion constraints of the robots, the presented approach features successive expansion of the sampling area, where increasingly larger sub-matrices of the original field are reconstructed analytically.

## **1.3** Overview of Contributions

The contributions of this dissertation are primarily on the complex dynamic modeling of tail-driven robotic fish, its related averaging and control theory, and an adaptive sampling framework using a group of robots. These contributions are further elaborated below.

First, the guidelines of robotic fish system design are provided and several types of robotic fish prototypes are developed for aquatic environmental monitoring, robot-animal interaction study, and education and outreach.

Second, a dynamic model for a tail-actuated robotic fish has been developed. In addition, a computationally efficient approach is proposed and validated for adapting the drag coefficient of the robotic fish based only on the tail-beat bias.

Third, to design and control robotic fish actuated with a flexible tail, it is essential to have a faithful and efficient dynamic model. Existing studies typically adopt a linear Euler-Bernoulli beam model is adopted to describe the beam dynamics; however, these models cannot accurately capture the beam dynamics when the beam undergoes large deformations. A model consisting of multiple, serially connected rigid segments is developed to describe the flexible beam, the effectiveness of which is validated with experimental results.

Fourth, a novel control-oriented data-driven averaging approach is proposed to approximate the original dynamics of robotic fish. The merit of the proposed approach, is demonstrated in the design of a hybrid controller for targeting tracking and in the application to study of steady turning.

Last, a school of robotic fish is proposed for adaptive sampling and reconstruction of aquatic fields. Distinguished from most existing work, we propose to expand the sampling area successively and reconstruct the field iteratively via analytical solutions. The effectiveness of the approach is validated via extensive simulations based on real data traces of the temperature field in

a large water tank. It shows that the proposed adaptive sampling solution is more computationally efficient and requires shorter travel distances for the robots than a competing matrix completion-based method.

## **1.4 Publications**

#### **1.4.1 Journal Articles**

- 1. J. Wang, X. Tan, "Averaging of Tail-Actuated Robotic Fish Dynamics through Force and Moment Scaling", *IEEE Transactions on Robotics*, under review, 2014
- 2. J. Wang, P. K. McKinley, X. Tan, "Dynamic modeling of robotic fish with a base-actuated flexible tail", *Journal of Dynamic Systems, Measurement, and Control*, to appear, 2013
- Y. Wang, R. Tan, G. Xing, X. Tan, J. Wang, R. Zhou, "Spatiotemporal aquatic field reconstruction using cyber-physical robotic sensor systems", ACM Transaction on Sensor Networks, vol. 10, no. 57, 2014
- Y. Wang, R. Tan, G. Xing, J. Wang, X. Tan, "Profiling aquatic diffusion process profiling using robotic sensor networks", *IEEE Transactions on Mobile Computing*, vol. 13, no. 4, pp. 880-893, 2014
- 5. J. Wang, X. Tan, "A dynamic model for tail-actuated robotic fish with drag coefficient adaptation", *Mechatronics*, vol. 23, no. 6, pp. 659-668, 2013

#### **1.4.2** Conference Proceedings

- C. Liu, S. Chen, J. Wang, X. Tan, "Study on shortest path planning for robotic fish actuated by a caudal fin" [in Chinese], *Proceedings of the 33rd Chinese Control Conference*, Nanjing, China, to appear, 2014
- Y. Wang, R. Tan, G. Xing, J. Wang, X. Tan, X. Liu, "Aquatic debris monitoring using smartphone-based robotic sensors", *Proceedings of the 13th ACM/IEEE International Conference on Information Processing in Sensor Networks*, Berlin, Germany, pp. 13-24, 2014
   [Acceptance rate: 21%; Runner-up for Best Paper Award]
- F. Zhang, J. Wang, J. Thon, C. Thon, E. Litchman, X. Tan, "Gliding robotic fish for mobile sampling of aquatic environments" (Invited), *Proceedings of 11th IEEE International Conference on Networking*, Sensing and Control, Miami, FL, pp. 167-172, 2014
- S. Chen, J. Wang, X. Tan, "Target-tracking control design for a robotic fish with caudal fin," *Proceedings of 32nd Chinese Control Conference*, Xi'an, China, pp. 844-849, 2013
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# Chapter 2

# Development of Robotic Fish and Their Applications

# 2.1 Introduction

Over the past two decades, there has been significant interest in the development of robotic fish. These low-cost, small form-factor platforms present a variety of constraints for design. The main design criteria in this work are to develop a robust system that is low-cost and easy to be manufactured, extended and serviced. The chapter is organized as follows. In Section 2.2, a detailed description of the network-robotic-fish is presented. In Section 2.3, a gliding robotic fish prototype and its application to environmental monitoring are described. In Section 2.4, a predator robotic fish and its application to robot-fish interaction are discussed. The use of robotic fish in education and outreach activities are briefly summarized in Section 2.5. These prototypes were developed jointly with several other members of the Smart Microsystems Lab.

## 2.2 Network-Robotic-Fish

A robotic fish prototype, called network-robotic-fish in this dissertation, is first described. The name comes from the intended purpose of this class of robotic fish for cooperative sensing as a mobile network. The design of such robots is faced with several challenges. Simplicity and ease of

fabrication are critical to keeping the cost of the robot node low, however, system robustness and modularity are also important considerations.

#### 2.2.1 Hardware structure

To address the challenges discussed above, we have developed a two-layer hardware structure for the robotic node with two micro-controllers. The navigation board is in charge of all the navigation-related functions, including drivers of sensors and actuator, and their coordinations. On the other hand, the processing board to the board takes care of higher-level information processing and decision-making tasks. The processing board also deals with communication and coordination among individual nodes. Such a design can significantly reduce the chances of conflict between time-critical components, such as the wireless radio and the navigation controller. More importantly, the hardware results in good extensibility and robustness properties for the design. The navigation board is relatively fixed in terms of low-level control of the robot and plenty of hardware interfaces are left for additional sensors. On the other hand, the processing board is designed to accommodate different applications.

The processing board is a duplicate of a standard Telosb mote, which features a MSP430 microcontroller and CC2420 radio stack. The layout, as shown in Fig. 2.1, of the the navigation board for network-robotic-fish accounts for interference isolation and minimization of total space.

#### 2.2.2 Software architecture

The robotic fish software architecture, shown in Fig. 2.2, consists of the firmware layer, the nodelevel module controllers layer, and customizable head-level computation and user-level application layer.





(b)

Figure 2.1: Navigation board

The novelty of this design is that we implement the software using two self-governed systems with an SPI interface between them. On the navigation board, a motion controller and sensor controller are developed upon the drivers for the hardware. On the processing board, a storage controller and network controller are implemented to control the data flow in the node and between the nodes, respectively. Furthermore, a user-level applications layer is developed to adapt different objectives.



Figure 2.2: Software architecture of network-robotic-fish.

#### 2.2.3 System assembly

One of the most time-consuming tasks in robotic fish development is the fabrication of shell with a streamlined shape with good hydrodynamic performance. In the first two generations of robotic fish, shown in Figs. 2.3(a) and 2.3(b) we have manually manufactured the shells. The mold has to be redesigned every time when evolving the robot's body. Another disadvantage of such an approach is that the body is not accessible<sup>1</sup>, which creates significant problems when it comes to diagnosis and service of the robot.

To accelerate the assembling process, an Objet Connex350 Multi-Material 3D printer is utilized to design and develop the robot shell. An important feature of this 3D printed platform is the selfcontainment of the system structure: the waterproof servo can be directly mounted to the bracket at the end portion; the *navigation board* is placed on the stand in the shell; extra capsules are left

<sup>&</sup>lt;sup>1</sup>The only way to access the robot's body is to cut off the shell and repair it afterwards, which is time-consuming and reduces the reliability.



(a)



(b)



Figure 2.3: Three generations of robotic fish: (a) First generation of robotic fish; (b) DO fish (second generation); (c) network-robotic-fish (third generation)

	Component name	Component model
1	Micro-controller ()	Microchip dsPIC6014A
2	Micro-controller (on TelosB)	TI MSP430
3	Wireless module (on TelosB)	TI CC2420
4	Battery	Batteryspace 7.4V (13.32Wh) Li-ion Polymer Battery Pack
5	Servo motor	Hitec Servo HS-5646WP (waterproof)
6	GPS	Garmin GPS 18x LVC
7	Gyro	ST LPY503AL
8	Accelerometer+Compass	ST LPY503AL
9	Temperature sensor	TMP36

Table 2.1: Selected components used in network robotic fish.

for the balance purposes. Any of these components can be easily detached and replaced within a hour or so, which highly improves the flexibility and extensibility of the system design, and also cuts the cost.

Table 2.1 lists the details of components on network robotic fish mentioned before in this section.

## 2.3 Gliding Robotic Fish

Gliding robotic fish is a new type of underwater vehicles, which is a combination of underwater glider and robotic fish. Other than the swimming motion by flapping the tail as robotic fish do, the robot has a mechanism of gliding by utilizing the difference between its gravity and buoyancy.

Fig. 2.4 illustrates the two gliding-based locomotion modes for the gliding robotic fish. The robot achieves a zig-zag motion by periodically varying its weight and the center of mass, as shown in Fig. 2.4(a). It can also realize a novel 3D spiral motion, shown in Fig. 2.4(b) by deflecting the tail during gliding.

We have successfully built a gliding robotic fish prototype, called GRACE (Gliding Robot



Figure 2.4: Illustration of gliding robotic fish motion profiles [2]. (a) Rectilinear gliding motion; (b) 3D spiraling.

ACE) [2,85], as shown in Fig. 2.5.

In November, 2012, we took GRACE, which was equipped with a crude oil sensor, to Kalamazoo River<sup>2</sup>, MI, to detect the crude oil content. In this test, GRACE swam along the river by flapping its tail as robotic fish and collected the data. The data obtained from the sensor were

<sup>&</sup>lt;sup>2</sup>Kalamazoo River was polluted by an oil spill in July 2010, when a pipeline burst and oil flowed into Talmadge Creek, a tributary of the Kalamazoo River.



Figure 2.5: Gliding robotic fish prototype developed at Smart Microsystems Lab. [2]

transmitted wirelessly to the base station with the location information from the GPS unit. Fig. 2.6 shows one sampling trajectory of GRACE.



Figure 2.6: Swimming trajectory of GRACE along the Kalamazoo River. [2]

Results from this section have appeared in the publication [2]. I was mainly responsible for system integration of GRACE and troubleshooting during the tests.

# 2.4 Predator Robotic Fish

We are also interested in using our robotic fish to study behaviors of live fish. Collaborating with Dr. Boughman's group at Michigan State University, we have developed a predator robotic fish to study the response behavior of stickleback fish as shown in Fig. 2.7.



(a)



Figure 2.7: Robot-fish interactions. (a) Predator robotic fish prototype; (b) live stickleback fish approaching the robot.

A C# code is implemented to capture the signal from a joystick and transfer it to the Xbee

module<sup>3</sup>, as shown in Fig.2.7(a). Users can control the robot's tail motion (i.e., beat-frequency and beat-bias) by operating the joystick. The robot is shaped and painted to mimic a trout, which serves as a predator to stickleback fish. Fig. 2.7(b) shows one frame of the video taken when a stickleback swam up tp the robot.

## 2.5 Eduction Activities and Outreach Using Robotic Fish

Aside from their vast potentials for research, robotic fish offer rich opportunities for reaching out to students and the general public. Bio-inspired robots are valuable and effective means for attracting students to the areas of science, technology, engineering and math (STEM) [86–88].

Robotic fish have been used as educational tools all over the world. For instance, the carp-like robots in [8,89] have attracted thousands of youth during their exhibits at London Aquarium [89].

We have presented our robotic fish at various outreach programs, including the first and second USA Science and Engineering Festivals, in 2010 and 2012, respectively. At these events, the visitors got a chance to not only see the robotic fish in action, but also interacted with our lab members to understand the technology and its applications, as shown in Fig. 2.8.

<sup>&</sup>lt;sup>3</sup>http://www.digi.com/xbee/.



(a)



Figure 2.8: Smart Microsystems Lab's robotic fish exhibit at the 2010 USA Science and Engineering Festival.

# Chapter 3

# A Dynamic Model for Tail-actuated Robotic Fish with Drag Coeff cient Adaptation

## 3.1 Introduction

Dynamic modeling [11, 12, 18, 19, 45, 47, 90–97], trajectory planning, and control [11, 38, 98–103] of robotic fish have received extensive attention. A major challenge in the modeling of robotic fish lies in properly capturing the fluid-structure interactions and the resulting force and moment on the robot. While computational fluid dynamics (CFD) modeling [32, 33, 104, 105] can provide high-fidelity representation of such interactions, it is not amenable to control design. A few alternatives are available. The first is to apply quasi-steady lift and drag models from airfoil theory to the body and fin surfaces of underwater robots [11, 90, 93, 94]. Another approach is to assume perfect fluids (irrotational potential flow) and exploit the symmetry to obtain a finite-dimensional model [95, 102], but some coefficients of such models may have to be obtained numerically [95]. Effects of vorticity can be accommodated by assuming, for example, vortices periodically shed from the tail fin [18, 92].

The third approach to the evaluation of hydrodynamic forces is to use Lighthill's elongatedbody theory [25]. This theory captures the reactive force between body and fluids with an addedmass effect, and it approximates the effect of wake dynamics by considering the momentum balance in a half-space control volume, which contains the deforming body and is bounded by a
vertical plane attached to the trailing edge. Consequently, the resulting force and moment only depend on the configuration of the (robotic) fish. Lighthill's model strikes a sound balance between fidelity and simplicity, and its effectiveness in robotic fish modeling has been demonstrated in [45, 106].

The contributions of the work reported in this chapter are two-fold. First, we present a dynamic model for a tail-actuated robotic fish by combining rigid-body dynamics with Lighthill's largeamplitude elongated-body theory, and validate the model with extensive experiments conducted on a robotic fish prototype. Specifically, we consider rigid-body dynamics for the robot body and adopt Lighthill's theory to capture the tail-actuation effect. To our best knowledge, such an approach has not been reported in the literature. Existing work [45, 106] involving Lighthill's theory was focused on steady-state motions, where the dynamics of the robot was not considered. Chen *et al.* applied a similar approach to evaluate the thrust produced by undulatory body motion of a robot tuna [107], but the analysis therein dealt with only point-mass dynamics for the robot, where forward-swimming was studied via simulation. Note that rigid body dynamics allows one to capture both body translation and rotation, and is significantly more complex and relevant than point-mass dynamics in terms of capturing the true behavior of the robot. Using the proposed model, we investigate the role of incorporating the body motion in evaluating the tail-produced hydrodynamic force, and show that ignoring the body motion (as often done in the literature [12, 14,45]) results in significant overestimate of the thrust force and robot speed.

The second contribution is a computationally efficient approach to the adaptation of drag coefficients for robotic fish. These coefficients play a significant role in the modeling of underwater vehicles and are often functions of the angle of attack [108, 109]. For a carangiform robotic fish, however, its body and/or fins constantly undergo large deformation or rigid-body motion, and consequently, the angle of attack oscillates due to the body recoil. For these reasons, modeling variable drag coefficients is challenging, and existing work on robotic fish modeling has predominantly adopted constant drag coefficients [12,93,110]. In this work, we propose a novel approach to adapting the drag coefficients. For ease of presentation, we focus on a robotic fish actuated by an oscillating tail, where the tail beat bias, frequency, and amplitude are the three kinematic parameters one can change. We first study the effect of each parameter on the average of the angle of attack, and find that the bias plays a much more significant role than the other two. We then propose determining the drag coefficients as simple polynomial functions of the tail beat bias. The effectiveness of the adaptation scheme is demonstrated through the comparison of simulation and experimental results, for both steady turning and transient dynamics in forward swimming.

The remainder of the chapter is organized as follows. First, the dynamic model is described in Section 3.2. Experimental validation of the model is presented in Section 3.3. The drag coefficient adaptation scheme is discussed in Section 3.4. Finally, concluding remarks are provided in Section 3.5.

# 3.2 The Dynamic Model for Robotic Fish

The robot is considered to consist of two parts, a body that does not deform and a moving or deforming tail. The motion of the robot body is governed by rigid-body dynamics with the addedmass effect incorporated. Lighthill's large-amplitude elongated-body theory is utilized to evaluate the hydrodynamic forces generated by the actuating tail (as opposed to the whole robot). We assume that the depth of the tail does not change abruptly along the length direction, thus meeting the "elongated body" requirement [25]. Although the proposed modeling approach is applicable for a general actuating tail, for ease of presentation, we focus on the case where the tail is rigid and actuated at its base. As mentioned in Section 3.1, one contribution of this work lies in the *combination* of the two elements, rigid-body dynamics and Lighthill's theory, for the dynamic modeling of robotic fish. While both elements are well known, for completeness of the chapter, brief review of each is provided in Sections 3.2.1 and 3.2.2.1, respectively.

### 3.2.1 **Rigid-body dynamics**

Fig. 3.1 shows the top view of the robotic fish, consisting of a rigid body and a tail. We follow [12] closely in describing the dynamics of the rigid body. [X, Y, Z] denote the inertial coordinates while [x, y, z] denote the body-fixed coordinates with unit vectors  $[\hat{x}, \hat{y}, \hat{z}]$ . We denote by  $\hat{m}$  and  $\hat{n}$  the unit vectors parallel and perpendicular to the tail, respectively. We assume that both the body and the tail are neutrally buoyant, and that, for the body, the center of gravity coincides with the center of geometry at point *C*. The velocity at *C* expressed in the body-fixed coordinates  $\vec{V}_C = [V_{Cx}, V_{Cy}, V_{Cz}]^T$  comprises surge  $(V_{Cx})$ , sway  $(V_{Cy})$  and heave  $(V_{Cz})$  components. In addition, the angular velocity  $\vec{\omega} = [\omega_x, \omega_y, \omega_z]^T$  comprises roll  $(\omega_x)$ , pitch  $(\omega_y)$  and yaw  $(\omega_z)$  expressed in the body-fixed coordinates. We use  $\alpha$  to denote the tail deflection angle with respect to the negative *x*-axis, and  $\beta$  to denote the angle of attack, formed by the direction of  $\vec{V}_C$  with respect to the *x*-axis. Finally,  $\psi$  denotes the heading angle, formed by the *x*-axis relative to the *X*-axis.

The linear momentum  $\vec{P}$  and angular momentum  $\vec{H}$  of the body in the body-fixed coordinates are expressed as

$$\vec{P} = \mathbf{M} \cdot \vec{V}_C + \mathbf{D}^T \cdot \vec{\omega}, \tag{3.1}$$

$$\vec{H} = \mathbf{D} \cdot \vec{V}_C + \mathbf{J} \cdot \vec{\omega}, \tag{3.2}$$

where  $\mathbf{M}$  and  $\mathbf{J}$  are the mass and inertia matrices, respectively, and  $\mathbf{D}$  is the Coriolis and centripetal matrix. For a rigid body in an inviscid fluid, Kirchhoff's equations of motion in the body-fixed



Figure 3.1: Top view of the tail-actuated robotic fish undergoing planar motion.

frame are [111, 112]

$$\vec{P} = \vec{P} \times \vec{\omega} + \vec{F}, \tag{3.3}$$

$$\vec{H} = \vec{H} \times \vec{\omega} + \vec{P} \times \vec{V}_C + \vec{M}, \tag{3.4}$$

where  $\vec{F} = [F_x, F_y, F_z]^T$ ,  $\vec{M} = [M_x, M_y, M_z]^T$  denote the external forces and moments about the body center *C*, respectively, and "×" denotes the vector product. The assumption of neural buoyancy implies that these forces and moments will only come from the hydrodynamic interactions between the robot and the fluid. We further assume that the body is symmetric with respect to the *xz*-plane and the tail moves in the *xy*-plane. Consequently, the heave velocity  $V_{Cz}$ , the roll rate  $\omega_z$ , and the pitch rate  $\omega_y$  are all equal to zero, in which case the system has three degrees of freedom, namely, surge ( $V_{Cx}$ ), sway ( $V_{Cy}$ ) and yaw ( $\omega_z$ ). We further assume that the inertial coupling between the surge, sway and yaw motions is negligible [12], implying  $\mathbf{D} = 0$ . Under these assumptions, following [111], (3.3) and (3.4) can be simplified as

$$(m_b - X_{\dot{V}_{Cx}})\dot{V}_{Cx} = (m_b - Y_{\dot{V}_{Cy}})V_{Cy}\omega_z + F_x,$$
(3.5)

$$(m_b - Y_{\dot{V}_{Cy}})\dot{V}_{Cy} = -(m_b - X_{\dot{V}_{Cx}})V_{Cx}\omega_z + F_y,$$
(3.6)

$$(J_{bz} - N_{\dot{\omega}_z})\dot{\omega}_z = (Y_{\dot{V}_{Cy}} - X_{\dot{V}_{Cx}})V_{Cx}V_{Cy} + M_z,$$
(3.7)

where  $m_b$  is the mass of the body, and  $J_{bz}$  is the inertia of the body about the *z*-axis.  $X_{\dot{V}_{Cx}}$ ,  $Y_{\dot{V}_{Cy}}$ and  $N_{\dot{\omega}_z}$  are the hydrodynamic derivatives that represent the effect of added mass/inertia on the body.

### **3.2.2** Evaluation of the hydrodynamic forces

To complete the model (3.5) - (3.7), we need to evaluate the hydrodynamic force  $(F_x, F_y)$  and moment  $M_z$ , which are due to both the tail actuation and the interaction of the body itself with the fluid. We first review Lighthill's large-amplitude elongated-body theory, and use it to evaluate the forces exerted on the tail (and transmitted to the rigid body) due to tail-fluid interactions. The drag and lift forces on the robotic fish body will then be described.

#### 3.2.2.1 Lighthill's large-amplitude elongated-body theory

Consider an elongated "body" moving in the *XY*-plane in water; here (and throughout Section 3.2.2.1) the term "body" could mean a live fish, a robotic fish, or in the context of our work, a flapping tail. As illustrated in Fig. 3.2, a frame of reference is chosen such that the water far from the body is at rest. The spinal column (analogy from a live fish) of the elongated body is assumed to be inextensible and is parameterized by  $\tau$ , with  $\tau = 0$  denoting the anterior end of the body and  $\tau = L$ 

denoting the posterior end. The coordinates  $(X(\tau,t), Y(\tau,t)), 0 \le \tau \le L$ , denote the trajectory of each point  $\tau$  on the spinal column at time t, which could be due to the body undulation/oscillation or the resulting translational/rotational motion of the body.



Figure 3.2: Illustration of the coordinate system for the spinal column of the elongated body (top view).

Following [25], given  $(X(\tau,t), Y(\tau,t))$ , the hydrodynamic reactive force density at each point  $\tau < L$ , due to the added-mass effect, is

$$\vec{f}(\tau) = \begin{pmatrix} f_X(\tau) \\ f_Y(\tau) \end{pmatrix} = -m \frac{\mathrm{d}}{\mathrm{d}t} (v_\perp \hat{n}), \qquad (3.8)$$

and at  $\tau = L$ , there is a concentrated force

$$\vec{F}_L = \begin{pmatrix} F_{LX} \\ F_{LY} \end{pmatrix} = \begin{bmatrix} -\frac{1}{2}mv_{\perp}^2\hat{m} + mv_{\perp}v_{\parallel}\hat{n} \end{bmatrix}_{\tau=L}.$$
(3.9)

The two terms in (3.9) account for the pressure force acting on  $\Pi$  and the force due to convection of momentum out of  $\mathscr{V}$  across  $\Pi$ , respectively, where  $\Pi$  is the plane at  $\tau = L$  that is perpendicular to  $\hat{m}$ , and  $\mathscr{V}$  is the half-space bounded by  $\Pi$  that includes the fish but excludes the wake [25]. In (3.8)

and (3.9), *m* denotes the virtual mass per unit length and can be approximated by  $\frac{1}{4}\pi\rho d^2$ , where  $\rho$  is the density of water and *d* is the depth of fish cross-section (in -Z direction) at  $\tau$  (and thus can vary with  $\tau$ ). As illustrated in Fig. 3.2,  $\hat{m} = (\partial X/\partial \tau, \partial Y/\partial \tau)^T$  and  $\hat{n} = (-\partial Y/\partial \tau, \partial X/\partial \tau)^T$  represent the unit vectors tangential and perpendicular to the spinal column, respectively, and  $v_{\parallel}$  and  $v_{\perp}$  represent the components of the velocity  $\vec{v} = (\partial X/\partial t, \partial Y/\partial t)^T$  at  $\tau$  in  $\hat{m}$  and  $\hat{n}$  directions, respectively:

$$v_{\parallel} = \langle \vec{v}, \hat{m} \rangle = \frac{\partial X}{\partial t} \frac{\partial X}{\partial \tau} + \frac{\partial Y}{\partial t} \frac{\partial Y}{\partial \tau}, \qquad (3.10)$$

$$v_{\perp} = \langle \vec{v}, \hat{n} \rangle = -\frac{\partial X}{\partial t} \frac{\partial Y}{\partial \tau} + \frac{\partial Y}{\partial t} \frac{\partial X}{\partial \tau}, \qquad (3.11)$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product of vectors. Note that the actual mass of the considered elongated-body is negligible (relative to the virtual mass) when the width of the body cross section is much thinner than the depth [25]. The latter typically holds true for the tail of a robotic fish.

#### 3.2.2.2 Hydrodynamic force due to an oscillating tail

Following Lighthill's large-amplitude elongated-body theory, we need to know the motion of every point along the tail over time, to evaluated the tail actuation-induced hydrodynamic force.

Refer to Fig. 3.1. The velocity  $\vec{V}_A$  at the body-tail joint A can be expressed as

$$\vec{V}_A = \vec{V}_C - \omega_z c \hat{y},\tag{3.12}$$

where *c* is the distance from the body center to the joint. The velocity at each point  $\tau$  along the tail is

$$\vec{V}_{\tau} = \vec{V}_C - \omega_z c \hat{y} + (\dot{\alpha} + \omega_z) \tau \hat{n}, \qquad (3.13)$$

where  $\dot{\alpha}$  denotes the angular velocity of the tail with respect to the body. Note here that  $\tau = 0$  would represent the point *A*. In (3.13),  $\vec{V}_{\tau}$  is calculated incorporating the robot body motion ( $\vec{V}_C$  and  $\omega_z$ ), which is often ignored in the literature [12, 14, 45]. In Section 3.3.4, we will discuss the role of incorporating the body motion in the appropriate evaluation of hydrodynamic forces.

Let  $\vec{V}_{\tau_{\perp}}$  be the projection of  $\vec{V}_{\tau}$  along the  $\hat{n}$  direction, i.e.,  $\vec{V}_{\tau_{\perp}} = \langle \vec{V}_{\tau}, \hat{n} \rangle \hat{n}$ . Using (3.8), we evaluate the hydrodynamic force density at  $\tau$  as

$$\vec{f}(\tau) = -m \frac{\mathrm{d}\vec{V}_{\tau_{\perp}}}{\mathrm{d}t}.$$
(3.14)

Then the hydrodynamic force on the tail due to the added-mass effect is

$$\vec{F}_{a} = \int_{0}^{L} \vec{f}(\tau) d\tau = F_{am} \,\hat{m} + F_{an} \,\hat{n}, \qquad (3.15)$$

where *L* is the length of the tail, and  $F_{am}$  and  $F_{an}$  are the components of  $F_a$  along  $\hat{m}$  and  $\hat{n}$ , respectively.

From (3.9), the concentrated force acting at the tail tip is evaluated as  $\vec{F}_c = F_{c1}\hat{m} + \vec{F}_{c2}\hat{n}$ , where

$$F_{c1} = -\frac{1}{2}mV_{L_{\perp}}^{2},$$
$$F_{c2} = mV_{L_{\perp}}V_{L_{\parallel}}.$$

The total hydrodynamic force acting on the tail is then

$$\vec{F}_h = F_{hx} \cdot \hat{x} + F_{hy} \cdot \hat{y},$$

where

$$F_{hx} = -(F_{am} + F_{c1})\cos\alpha + (F_{an} + F_{c2})\sin\alpha, \qquad (3.16)$$

$$F_{hy} = -(F_{am} + F_{c1})\sin\alpha - (F_{an} + F_{c2})\cos\alpha.$$
(3.17)

The hydrodynamic force-induced moment relative to the center C of the robot body is

$$\vec{M}_h = \int_0^L \vec{r}_{C\tau} \times \vec{f}(\tau) \,\mathrm{d}\,\tau + \vec{r}_{CL} \times \vec{F}_c, \qquad (3.18)$$

where  $\vec{r}_{C\tau}$  denotes the vector from the body center *C* to the point  $\tau$  on the tail, i.e.,  $\vec{r}_{C\tau} = -(c + \tau \cos \alpha) \hat{x} - (\tau \sin \alpha) \hat{y}$ . It is clear that all vectors in (3.18) lie in the *xy*-plane, and thus the only non-zero component of  $\vec{M}_h$  is along the *z*-direction, which we denote as  $M_{hz}$ .

#### 3.2.2.3 Drag and lift on the body

As shown in Fig. 3.1, besides the hydrodynamic force and moment transmitted from the tail, the robot body also experiences drag force  $F_D$ , lift force  $F_L$ , and drag moment  $M_D$  [11, 12, 93]:

$$F_D = \frac{1}{2}\rho |V_C|^2 SC_D,$$
(3.19)

$$F_L = \frac{1}{2}\rho |V_C|^2 S C_{L\beta}\beta, \qquad (3.20)$$

$$M_D = -K_D \omega_z^2 \operatorname{sgn}(\omega_z). \tag{3.21}$$

where *S* is a suitably defined reference surface area for the robot body.  $C_D$  and  $K_D$  are the drag force coefficient and drag moment coefficient, respectively.  $C_{L\beta}$  is the lift coefficient per angle of attack  $\beta$ . The drag coefficients, as we will discuss later, are generally dependent on the angle of

attack  $\beta$  of the robot body.

Finally, by adding the hydrodynamic forces and moments from the tail and directly on the body, we obtain  $F_x$ ,  $F_y$ , and  $M_z$  in (3.5) – (3.7) as

$$F_x = F_{hx} - F_D \cos\beta + F_L \sin\beta, \qquad (3.22)$$

$$F_y = F_{hy} - F_D \sin\beta - F_L \cos\beta, \qquad (3.23)$$

$$M_z = M_{hz} + M_D. \tag{3.24}$$

Note that,  $F_{hx}$ ,  $F_{hy}$  and  $M_{hz}$  contain the derivative of  $V_{Cx}$ ,  $V_{Cy}$ , and  $\omega_z$  since the robotic fish body's motion is incorporated in the evaluation of hydrodynamic forces at the tail section. With fundamental transformations, Eq. (5.1) could be re-written with  $V_{Cx}$ ,  $V_{Cy}$  and  $\omega_z$  on the right side of the equations and their time derivative on the left side to avoid the numerical loop in the simulation.

### 3.3 Experimental Model Validation

### 3.3.1 Experimental setup

To validate the proposed dynamic model in Section 3.2, experiments have been conducted with a free-swimming robotic fish prototype as shown in Fig. 3.3. The robot has a simple mechanism for actuation, and it satisfies most of the assumptions used in the modeling work, which facilitates model validation. Through a chain transmission mechanism, a servomotor (HS-5085MG from Hitec) was used to control the angular position of the tail shaft and thus the tail deflection angle  $\alpha$ . The tail was a rectangular carbon-fiber foil, which was 8 cm long, 2.5 cm wide (deep), and 1.1 mm thick. With an onboard microcontroller, the servomotor was programmed to rotate the tail

according to

$$\alpha(t) = \alpha_0 + \alpha_A \sin(\omega_\alpha t + \phi_\alpha) \tag{3.25}$$

with  $\alpha_0$ ,  $\alpha_A$  and  $\omega_\alpha$  denoting the bias, amplitude, and frequency of the tail beat, respectively; and  $\phi_{\alpha}$  denoting the phase that indicates the initial condition. When the bias  $\alpha_0 = 0$ , the robot performs forward swimming. When  $\alpha_0 \neq 0$ , the trajectory of the robot converges to a circular orbit. For model validation, we measured the turning radius (radius of the circular orbit) and turning period (time for completing a turn) during steady turning, under different combinations of  $\alpha_0$ ,  $\alpha_A$ , and  $\omega_{\alpha}$ . The experiments were conducted in an indoor kiddle pool (as shown in Fig. 7 of [106]), and the turning radius and period were measured with a timer and yardstick, respectively. Specifically, for each experiment, we would first wait until the robot entered steady turning, and then place a yardstick (suspended slightly higher than the top of the robot) along the line connecting the visually determined leftmost and rightmost points of the trajectory. Afterwards (while the robot continued turning) we would mark down the points A and B where the robot entered and left, respectively, the half-space defined by the yardstick. These two points were typically close to the visually determined left/rightmost points mentioned earlier. The distance between A and B and the difference between the times when the robot hit these two points (captured by a timer) were used to determine the turning radius and period, respectively. With any prescribed open-loop control pattern, the observed circular orbit shifted slightly over time, which could be due to the flow disturbance created by the swimming robot in the relatively small pool (diameter 1.75 m). To mitigate the impact of such shift on the measurement error, for each pattern, experiments were repeated for 6 times (restarting the robot from rest every time), from which the mean and standard deviation of the turning radius/period were evaluated. We note that, while the turning experiments conducted in this work did offer valuable insight on how tail-beat parameters could impact turning behavior, their primary use is in providing data for model validation under a wide range of openloop inputs. This is different from [113], where the authors focused on turning control strategies for multi-link robotic fish.



Figure 3.3: A free-swimming robotic fish prototype used for model validation.

### 3.3.2 Parameter identif cation

The parameters in simulation are measured directly or calculated based on measurement as shown in Table 3.1. The body inertia about the *z*-axis is evaluated as  $J_{bz} = \frac{1}{12}m_b(2c)^2$ . The added masses and inertia are calculated by approximating the robot body as a prolate spheroid [12, 112].

The drag and lift coefficients  $C_D$ ,  $C_{L\beta}$ , and  $K_D$  can be potentially obtained by performing tests in a water tunnel or from the CFD simulation, while in this work, we choose a more efficient way to empirically identify these coefficients. In particular, we have tuned these parameters to match the turning radius, turning period, and the average of  $|\beta|$  obtained in simulation with the experimental measurement under a particular tail beat pattern, with bias  $\alpha_0 = 20^\circ$ , amplitude  $\alpha_A = 15^\circ$ , and frequency  $\omega_A = 1.8\pi$  rad/s (0.9 Hz). The resulting coefficients are  $C_D = 0.2231$ ,  $C_{L\beta} = 4.50$ , and

Component	Parameter	Value
Body	mass $(m_b)$	0.311 kg
	distance from body center to the shaft $(c)$	0.07 m
	nominal body surface area (S)	$0.0108 \text{ m}^2$
	inertia $(J_{bz})$	$5.0797 \times 10^{-4} \text{ kg} \cdot \text{m}^2$
	$-X_{\dot{V}_{Cr}}$	0.0621 kg
	$-Y_{\dot{V}_{Cv}}$	0.2299 kg
	$-N_{\dot{\omega}_z}$	$1.0413 \times 10^{-4} \text{ kg} \cdot \text{m}^2$
Tail	tail length (L)	0.08 m
	tail depth $(d)$	d = 0.025m
	water density ( $\rho$ )	$1 \times 10^3 \text{ kg/m}^3$

Table 3.1: Parameter	values for	r simulations.
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 $K_D = 11.29 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ . These parameters are then used in independent model validation for all other tail beat patterns.

For a fully submerged body taking a shape of a prolate spheroid, which we will assume in this work, the added-mass terms can be evaluated analytically. Let the profile of the body be described by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1,$$
(3.26)

where a and b are the semi-axis length. In particular, considering the robotic fish body volume and geometric distribution, the values of a and b are set to 7 and 3.3 cm, respectively. Then the added mass terms are given by

$$X_{\dot{V}_{Cx}} = -k_1 m_f$$

$$Y_{\dot{V}_{Cy}} = -k_2 m_f,$$

$$N_{\dot{\omega}_z} = -k_3 J_{fz}$$
(3.27)

where positive constant  $k_1$ ,  $k_2$  and  $k_3$  are Lamb's k-factors dependent on the geometry of the body,

defined using two constant  $\alpha_0$  and  $\beta_0$ :

$$k_{1} = \frac{\alpha_{0}}{2 - \alpha_{0}}, \alpha_{0} = \frac{2(1 - e^{2})}{e^{3}} (\frac{1}{2} ln \frac{1 + e}{1 - e} - e),$$

$$k_{2} = \frac{\beta_{0}}{2 - \beta_{0}}, \beta_{0} = \frac{1}{e^{2}} - \frac{1 - e^{2}}{2e^{3}} ln \frac{1 + e}{1 - e},$$

$$k_{3} = \frac{e^{4} (\beta_{0} - \alpha_{0})}{(2 - e^{2})[2e^{2} - (2 - e^{2})(\beta_{0} - \alpha_{0})]},$$
(3.28)

where  $e = \sqrt{1 - (\frac{b}{a})^2}$ , designates the eccentricity of the ellipsoid. Furthermore, in Eq. (3.27), the mass of the displaced fluid is evaluated using  $m_f = 4/3\rho\pi ab^2$ , which is 0.3193 kg and the moment of inertia of the fluid's spheroid about the *z*-axis is  $J_{fz} = 1/5m_f(a^2 + b^2)$ .

### 3.3.3 Results on model validation

Figs. 3.4 – 3.6 show the comparison between the model prediction and experimental measurement of the turning radius and turning period, when we vary the tail beat bias  $\alpha_0$ , amplitude  $\alpha_A$ , and frequency  $\omega_{\alpha}$ , respectively, while holding the other two parameters constant.

Experimental results in Fig. 3.4 show that both the turning radius and the turning period drop monotonically with an increasing tail beat bias; in other words, the robot makes tighter and quicker turns as the bias is increased. For these experiments, the tail beat amplitude and frequency are held fixed at  $15^{\circ}$  and 0.9 Hz, respectively. From Fig. 3.4, these trends are well captured by the model prediction, which closely matches the experimental measurement.

Fig. 3.5 shows how the turning radius and period depend on the tail beat amplitude, where the bias and frequency are set to be  $20^{\circ}$  and 0.9 Hz, respectively. The model predicts that the turning radius is nearly constant (about 27.5 cm) over the tested range for the tail beat amplitude. The mean of the measured radii for each given amplitude stays within the band [26, 30.5] cm, which is



Figure 3.4: Comparison between model prediction and experimental measurement of (a) turning radius and (b) turning period as a function of the tail beat bias  $\alpha_0$ . The amplitude and frequency of the tail beat are fixed at 15° and 0.9 Hz, respectively.

close to the predicted value. The turning period is predicted by the model to decrease (i.e., faster turning) as the tail beat amplitude increases, which is supported by the experimental data both qualitatively and quantitatively.

The impact of the tail beat frequency on the turning behavior is shown in Fig. 3.6, where the chosen tail beat bias and amplitude are  $20^{\circ}$  and  $15^{\circ}$ , respectively. The model predicts that the turning radius has negligible dependence on the tail frequency, while the turning period drops with the frequency. As seen in Fig. 3.6, both trends are well supported by the experimental data.

# 3.3.4 Hydrodynamic forces with/without incorporating robotic f sh body motion

It is often tempting to consider the body as anchored when evaluating fin-produced hydrodynamic forces [12, 14, 45], since this would greatly simplify the model. It is of interest to assess the validity of such simplifications for a free-swimming robot where the robot body undergoes movement. We will use the proposed model in Section 3.2, which has proven effective as shown in the previous subsection, to evaluate the necessity of incorporating the body motion, by comparing the simulation results when such motion is and is not considered.

For this study, we take the tail beat bias  $\alpha_0$  to be 0, and thus the robot will perform forward swimming at the steady state. We will evaluate the impact of incorporating (or neglecting) body motion on the computed average thrust (over a tail-beat cycle) and the robot swimming speed. The thrust force is evaluated via

$$F_{thrust} = F_{hx}\cos(\psi - \psi_0) - F_{hy}\sin(\psi - \psi_0), \qquad (3.29)$$

where we recall from Fig. 3.1 that  $\psi$  denotes the heading of the robotic fish with respect to the



Figure 3.5: Comparison between model prediction and experimental measurement of (a) turning radius and (b) turning period as a function of the tail beat amplitude. The bias and frequency of the tail beat are fixed at  $20^{\circ}$  and 0.9 Hz, respectively.



Figure 3.6: Comparison between model prediction and experimental measurement of (a) turning radius and (b) turning period as a function of the tail beat frequency. The bias and amplitude of the tail beat are fixed at  $20^{\circ}$  and  $15^{\circ}$ , respectively.

*X*-axis in the inertial frame. In (3.29),  $\psi_0$  denotes the average of  $\psi$  over a tail-beat cycle at the steady state, while  $F_{hx}$  and  $F_{hy}$  are evaluated using (3.16) and (3.17). Ignoring the body motion in hydrodynamic force evaluation implies that the velocity  $\vec{V}_{\tau}$  at point  $\tau$  on the tail is changed from (3.13) to

$$\vec{V}_{\tau} = \dot{\alpha}\tau\hat{n}.\tag{3.30}$$

Fig. 3.7 compares the calculated average thrusts under different tail-beat amplitudes and frequencies, when the body motion is and is not incorporated in evaluating  $\vec{V}_{\tau}$ , respectively. It shows clearly that ignoring the body motion would result in significant over-estimate of the thrust. For example, when the tail beats at 0.9 Hz with an amplitude of 15°, the average thrust is 1.188 mN when the body motion is ignored, which is 57% larger than the value of 0.755 mN when the body motion is considered. Fig. 3.8 further compares the computed average swimming speeds under the two modeling schemes, from which we can see that ignoring the body motion results in an over-estimate of steady-state speed by about 25%. These results illustrate the profound impact of the body motion on the tail-generated hydrodynamic force.

## **3.4 Eff cient Adaptation of Drag Coeff cients**

### 3.4.1 Bias-based adaptation of drag coeff cients

The drag force and moment coefficients  $C_D$  and  $K_D$  in (3.19) and (3.21) have been assumed to be constants. Most (if not all) modeling work on robotic fish has assumed constant drag coefficients [12, 93, 94, 110]. However, it is known in fluid mechanics that the angle of attack influences the drag on a body that is moving through a fluid [11, 94, 108, 109]. For robotic fish, one challenge is that the angle of attack oscillates constantly because of the recoil motion of the body in response to



Figure 3.7: Simulated average thrust force with/without incorporating the body motion: (a) Thrust force versus tail-beat amplitude (frequency fixed at 0.9 Hz); (b) thrust force versus tail-beat frequency (amplitude fixed at  $15^{\circ}$ .



Figure 3.8: Simulated average velocity with/without incorporating the body motion: (a) Velocity versus tail-beat amplitude (frequency fixed at 0.9 Hz); (b) velocity versus tail-beat frequency (amplitude fixed at  $15^{\circ}$ .

tail movements. Furthermore, evaluation of the angle of attack involves a highly nonlinear function of the velocity components,  $\beta = \arctan(V_{Cy}/V_{Cx})$ . In this section, we present an approach to the efficient adaptation of drag coefficients for robotic fish, to maintain or even improve model fidelity without the need to explicitly evaluate the angle of attack. The word "efficient" here refers to computational efficiency (or low modeling complexity), as we will show in the later part of this section that such a varying drag effect can be well captured with a simple function of the tail-beat bias.

At the core of our approach is the exploitation of the close correlation between the angle of attack and the tail beat bias. Recall the form of the tail beat angle  $\alpha$  in (3.25). While the bias  $\alpha_0$ , amplitude  $\alpha_A$ , and frequency  $\omega_{\alpha}$  all could be varied over time (for example, when one controls the robot to follow some arbitrary path), such variations typically take place at a much slower time scale than the tail beat motion – a tail beat pattern will be repeated for at least several cycles before it is updated. Figs. 3.4–3.6 suggest that the turning radius strongly depends on the bias  $\alpha_0$ , but is nearly independent of the amplitude  $\alpha_A$  and frequency  $\omega_{\alpha}$ . The turning radius is intimately related to the angle of attack. To explicitly reveal the relationships between the angle of attack and the tail-beat pattern parameters, we have used the model identified in Section 4.5 to numerically evaluate the average  $\beta_0$  of the absolute value  $|\beta|$  of the angle of attack over each beat cycle, i.e.,

$$\beta_0 = \frac{\omega_\alpha}{2\pi} \int_{t_0}^{t_0 + 2\pi/\omega_\alpha} |\beta(t)| \mathrm{d}t.$$
(3.31)

From Fig. 3.9,  $\beta_0$  shows nearly linear dependence on the tail-beat bias angle, with little dependence on the amplitude or the frequency.

Based on the observations from Fig. 3.9 and on the relationships between the drag coefficients and the angle of attack [108], we propose adapting these coefficients using the tail-beat bias  $\alpha_0$ 



Figure 3.9: Simulation results on the dependence of the angle of attack on tail beat parameters, where  $\beta_0$  is the cycle-averaged absolute value of the angle attack: (a)  $\beta_0$  versus the tail beat bias  $\alpha_0$ ; (b)  $\beta_0$  versus the tail beat amplitude  $\alpha_A$ ; (c)  $\beta_0$  versus the tail beat frequency  $\omega_{\alpha}$ . In (a), the amplitude and frequency are fixed at 15° and 0.9 Hz; in (b), the bias and the frequency are fixed at 20° and 0.9 Hz; in (c), the bias and the amplitude are fixed at 20° and 15°.

(note that the underlying assumption here is that, the impact of the angle of attack,  $\beta$ , on drag can be well captured through the cycle-averaged absolute value of  $\beta$ ,  $\beta_0$ ):

$$C_D = C'_{D0} + C'_{D1} \alpha_0^2, \qquad (3.32)$$

$$K_D = K'_{D0} + K'_{D1}\alpha_0, \tag{3.33}$$

where  $C'_{D0}$ ,  $C'_{D1}$ ,  $K'_{D0}$  and  $K'_{D1}$  are constants that are empirically identified as follows. First, as described in Section 3.3.2, we seek values of  $C_D$ ,  $K_D$ , and  $C_{L\beta}$  such that the simulated turning radius and period best match the experimental results when the tail beats at 0.9 Hz with amplitude of 15° and bias of 20°. We then maintain the value of  $C_{L\beta}$ , and find the best parameter estimates for  $C_D$  and  $K_D$  when the bias is 30° and 40°, respectively. We have deliberately left out the cases of 25° and 35° biases at this stage, so that those data can be used in independent model validation. Table 3.2 lists the identified  $C_D$  and  $K_D$  identified for the three biases. We then use the values in Table 3.2 to identify  $C'_{D0}$ ,  $C'_{D1}$ ,  $K'_{D0}$  and  $K'_{D0}$  in (3.32) and (3.33) through the least-squareerror fitting. The resulting parameters are  $C'_{D0} = 0.1936$ ,  $C'_{D1} = 0.1412/\text{rad}^2$ ,  $K'_{D0} = 9.655 \times$  $10^{-4} \text{ kg} \cdot \text{m}^2$  and  $K'_{D1} = 4.383 \times 10^{-4} \text{ kg} \cdot \text{m}^2/\text{rad}$ , respectively.

Fig. 3.10 shows the comparison between the measured turning radii and periods and the values predicted by the model with drag coefficient adaptation. We can see that, with the bias-based adaptation of drag coefficients, the match between the model prediction and the experimental data is further improved over that in Fig. 3.4.

Table 3.2: Identified drag force and moment coefficients for different tail beating biases.

Bias $\alpha_0$ (°)	$C_D$	$K_D (\mathrm{kg}\cdot\mathrm{m}^2)$
20	0.2231	$1.129 \times 10^{-3}$
30	0.2112	$1.174 \times 10^{-3}$
40	0.2712	$1.282 \times 10^{-3}$



Figure 3.10: Comparison between experimental measurement of (a) turning radius and (b) turning period with those predicted by the model, where the drag coefficients are adapted and the lift force is evaluated using (3.20). The amplitude and frequency of the tail beat are fixed at  $15^{\circ}$  and 0.9 Hz, respectively.

To further validate the approach for drag coefficient adaptation, we have conducted additional experiments involving transients in forward swimming. A yellow LED powered by a button cell was taped to the top of the robot as a marker. The weight of the LED/cell add-on was 3 g, so its impact on the total mass (over 300 g) and inertia of the robot was negligible. Starting from at rest, the robot was programmed to swim forward (with the tail beat bias  $\alpha_0 = 0^\circ$ ) with the amplitude and frequency of the tail-beat equal to  $15^{\circ}$  and 0.9 Hz, respectively. We video-recorded the motion of the LED (and thus of the robot) with the ambient light turned off, and extracted the robot's time-trajectory by applying background subtraction to the images. The dynamic model has been simulated with the adapted and non-adapted drag coefficients, respectively. For the model with non-adapted drag coefficients,  $C_D = 0.2231$ ,  $K_D = 11.29 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ , which are identical to the coefficients obtained from the turning experiments as presented in Section 3.3.2. For the model with adapted drag coefficients,  $C_D = 0.1936$ ,  $K_D = 9.655 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ , which are evaluated based on (3.32) and (3.33), with  $\alpha_0 = 0$ . Fig. 3.11(a) and (b) compare the model predictions of the X/Y-coordinate time-trajectories of the robot with the experimental measurement, and Fig. 3.11(c) provides the comparison on the robot path in the XY-plane. It can be seen that the model with adapted drag coefficients outperforms the one with non-adapted coefficients, and is able to capture well both the transient and steady-state behaviors of the robot.

### **3.4.2** Elimination of the angle of attack from the lift force term

Note that, while the drag coefficients in (3.32) and (3.33) do not involve the angle of attack  $\beta$ , the evaluation of the lift force for the robot body in (3.20) still requires knowing  $\beta$ . In order to eliminate  $\beta$  from the overall model and thus reduce the model complexity, in this subsection we



Figure 3.11: Comparison between model predictions and experimental measurement for forward swimming (including transients): (a) time trajectory of *X*-coordinate of the robot; (b) time trajectory of the *Y*-coordinate of the robot; (c) path of the robot in the *XY*-plane. For both the experiment and simulations, the amplitude and frequency of the tail beat are fixed at  $15^{\circ}$  and 0.9 Hz, respectively.

explore replacing  $\beta$  in (3.20) with the tail-beat bias  $\alpha_0$ :

$$F_L = \frac{1}{2}\rho |V_C|^2 SC'_{L\beta} \alpha_0, \qquad (3.34)$$

where,  $C'_{L\beta}$  is the new lift coefficient associated with the tail bias. To identify  $C'_{L\beta}$ , we tune  $C'_{L\beta}$ ,  $C_D$ , and  $K_D$  so that the simulated turning radius, turning period, and average of  $|\beta|$  match those obtained in experiments when the tail beats with bias  $\alpha_0 = 20^\circ$ , amplitude  $\alpha_A = 15^\circ$ , and frequency  $\omega_A = 1.8\pi$  rad/s. The estimates are  $C'_{L\beta} = 0.935$ ,  $C_D = 0.2131$ , and  $K_D = 1.144 \times 10^{-3}$  kg·m<sup>2</sup>. With  $C'_{L\beta}$  fixed, we can then follow the similar steps as in Section 3.4.1 to determine the drag coefficient adaptation laws (3.32) and (3.33). Note that the four parameters in (3.32) and (3.33) are identified twice since we want to compare the two cases when the evaluation of the lift force involves the angle of attack  $\beta$  or its surrogate  $\alpha_0$ . In practice, one only needs to identify these parameters once given the choice of (3.20) or (3.34) for the lift force evaluation.

Fig. 3.12 and Fig. 3.13 show the model prediction performance when drag coefficients are fixed and adapted, respectively, where the lift force is evaluated using (3.34) instead of using (3.20) as for Figs. 3.4-3.6 and 3.10. From Fig. 3.12, the model with the modified lift term (3.34) is still able to capture the trends of how the turning radius and period vary with the tail bias, although the match has degraded from that seen in Fig. 3.4. From Fig. 3.13, one can see that, with the drag adaption, significantly better match between the model prediction and experimental data is achieved.

### 3.5 Conclusion

In this work, we have presented a complete dynamic model for robotic fish actuated by a caudal fin, where Lighthill's elongated-body theory is used to evaluate the tail-generated hydrodynamic



Figure 3.12: Comparison between experimental measurement of (a) turning radius and (b) turning period with those predicted by the model, where the drag coefficients are not adapted and the lift force is evaluated using (3.34). The amplitude and frequency of the tail beat are fixed at  $15^{\circ}$  and 0.9 Hz, respectively.



Figure 3.13: Comparison between experimental measurement of (a) turning radius and (b) turning period with those predicted by the model, where the drag coefficients are adapted and the lift force is evaluated using (3.34). The amplitude and frequency of the tail beat are fixed at  $15^{\circ}$  and 0.9 Hz, respectively.

force. For experimental validation purpose, we have considered a rigid tail with biased sinusoid oscillation and treated the turning radius and turning period as outputs in the comparison between model prediction and experimental results. Additional model validation has also been conducted with experimental data of forward swimming including the transients. We have found that, incorporating the body motion in the evaluation of hydrodynamic forces plays a significant role in capturing the dynamics. We have also shown that, the drag force and moment coefficients are strongly dependent on the tail-beat bias, and subsequently we have proposed a novel scheme to adapt these coefficients and lift force based on the tail movement pattern. The effectiveness of this approach has been further demonstrated in experiments.

# Chapter 4

# Dynamic Modeling of Robotic Fish with a Base-actuated Flexible Tail

# 4.1 Introduction

Numerous designs of actuation mechanisms have been proposed for robotic fish [5, 11, 12, 19, 24, 35–39, 41, 43–46, 49, 114]. A typical approach is to use multiple links actuated separately or jointly to deform the body itself [5, 8, 16, 19, 115, 116], which requires multiple actuators and/or complex transmission mechanisms. Another class of designs involves an oscillating caudal fin (e.g., [6,11,12,44,45]), sometimes in conjunction with pectoral fins [11]. Among various designs, tail actuation is especially attractive since it is easy to realize, enables both forward swimming and turning maneuvers, and leaves the majority of the body free of moving parts. The latter is important when the robot is used in applications such as mobile sensing, where the body space can be maximally used to house sensors and electronics [1]. Many tail-actuation designs involve rigid, oscillating plates [11,14,15,93]. This design lends itself to simple construction and tractable analysis. However, it has been recognized that the flexibility of body and fin structures has a pronounced impact on the swimming performance of biological and robotic fish [18, 117, 118]. Flexible caudal fins can be realized by motor-driven compliant beams or plates [19, 44], or directly through soft actuation materials such as ionic polymer-metal composites (IPMCs) [6, 9, 12, 45]. While these active materials possess intriguing properties, the thrust they can produce is still relatively weak and the long-term repeatability of their actuation behavior is yet to be established. Therefore, base-actuated soft, passive structures remain a competitive option for flexible tails.

To design and control robotic fish actuated with a flexible tail, it is essential to have a faithful and efficient dynamic model. Modeling of a flexible beam attached to a moving base in the air has been studied by a number of researchers [119–121]. However, in this work, a major challenge in the modeling lies in properly capturing the fluid-structure interactions and the resulting force and moment on the robot. Most existing work on modeling has dealt with rigid fins [11, 14, 15, 90, 92, 93], although modeling of flexible fins has been conducted recently by several groups [12, 19, 45, 122]. In [19], Alvarado et al. focused on designing compliant bodies to achieve biomimetic locomotion efficiency and maneuverability. To demonstrate the results, they also compared the experimental results with the linear beam model. Researchers have also developed models that capture the actuation physics of an anchored IPMC beam and the complex hydrodynamic interactions between IPMC and fluid [12, 45]. In [122], Kopman et al. described a modular biomimetic robotic fish developed for educational activities, along with a modeling framework for predicting the robot's static thrust production. In all these studies, a linear Euler-Bernoulli beam model was adopted to describe the beam dynamics. A disadvantage of this approach is that, under large oscillations, these models do not accurately capture the beam dynamics [123–125] and consequently the hydrodynamics.

In this work, we take a significant departure from the beam model approach by approximating a flexible tail with multiple rigid segments connected in series through torsional springs and dampers. A similar multi-segment approach has been used to model a beam under large deformation in air [125,126] and model a flexible beam in flow sensing [127]. However, the multi-segment beam model has not been explored in the modeling of a base-actuated flexible beam subject to complex hydrodynamic interactions. For comparison, we also present a model based on linear beam theory, as widely adopted in other studies [12, 19, 45, 122]. Note that, for ease of presentation, a tail with uniform height is considered in this work. Despite its simple appearance, this case captures the key essential challenges (nonlinear beam dynamics under large deformation, coupled with hydrodynamic interactions) that would be present for cases with more general tail shapes. We evaluate the hydrodynamic force on the actuating tail using Lighthill's large-amplitude elongated-body theory [25], because it has a sound balance between fidelity and simplicity and its effectiveness in robotic fish modeling has been demonstrated in our prior work [15, 45]. A weakness of Lighthill's theory is that it neglects the effect of vortex wake on the pressure distribution on the body and thus neglects the effect of vortex shedding on the thrust production [128]. Incorporating such vortex-shedding effects (for example, using the vortex ring panel method [129]), however, typically requires CFD simulation and is not amenable to the development of a model that is suitable for robot optimization and control, which is the goal of this work.

To compare and validate the models, we conduct extensive experiments with a robotic fish prototype. We find that both models have similar predictions that are close to the experimental measurements when the flexible beam is under small deformations. However, the discrepancy between the predictions becomes greater as the beam experiences larger deformations. In particular, we show that the multi-segment model is able to predict precisely the transient trajectory and steady-state speed of the free-swimming robot, as well as the dynamic shape of the tail, under a wide range of actuation inputs. In comparison, the linear beam model-based approach is only able to capture the robot speed and tail shape when the tail undergoes relatively small deformations.

The remainder of the chapter is organized as follows. The overall dynamic model for a tailactuated robotic fish is first reviewed in Section 4.2, where the hydrodynamic forces generated by the tail are evaluated by Lighthill's theory. The use of Lighthill's theory requires knowing the tail shape as a function of time, which is the main focus of this chapter. The Euler-Bernoulli beam-based model and the multi-segment model for the based-actuated flexible tail are developed in Sections 4.3 and 4.4, respectively. Experimental validation and comparison of the models are presented in Section 4.5. Finally, concluding remarks are provided in Section 4.6.

### 4.2 The Model Framework for Tail-actuated Robotic Fish

The robot is assumed to comprise two parts, a rigid body and a flexible tail. The motion of the robot body is governed by rigid-body dynamics with the added-mass effect incorporated. Lighthill's large-amplitude elongated-body theory is adopted to evaluate the hydrodynamic forces generated by the flexible tail's motion. We assume that the height of the tail does not vary abruptly along its length, thus meeting the "elongated body" requirement [25].

Fig. 4.1 shows a schemata of the robotic fish, with [X, Y, Z] denoting the inertial coordinates, and [x, y, z] denoting the body-fixed coordinates with unit vectors  $[\hat{x}, \hat{y}, \hat{z}]$ . We denote by  $\hat{m}$  and  $\hat{n}$  the unit vectors parallel and perpendicular to the tail, respectively. We assume that both the body and the tail are neutrally buoyant, and that the center of gravity of the body coincides with the center of geometry at point *C*. The velocity at *C* expressed in the body-fixed coordinates  $\vec{V}_C = [V_{Cx}, V_{Cy}, V_{Cz}]^T$  comprises surge  $(V_{Cx})$ , sway  $(V_{Cy})$  and heave  $(V_{Cz})$  components. In addition, the angular velocity  $\vec{\omega} = [\omega_x, \omega_y, \omega_z]^T$  comprises roll  $(\omega_x)$ , pitch  $(\omega_y)$  and yaw  $(\omega_z)$  expressed in the body-fixed coordinates. We use  $\alpha$  to denote the tail deflection angle (the tangential direction of the flexible tail at the base) with respect to the negative *x*-axis, and  $\beta$  to denote the angle of attack, formed by the direction of  $\vec{V}_C$  with respect to the *x*-axis. Finally,  $\psi$  denotes the heading angle, formed by the *x*-axis relative to the *X*-axis.

We note that the actual mass of the tail is negligible (relative to the virtual mass) when the width of the tail is much thinner than the height [25], which is typically true for the tail of a robotic



Figure 4.1: Schematic representation of the robotic fish in planar motion.

fish. For example, in the model validation part of this work, we use a flexible tail with a height of 2.5 cm and a thickness of 0.3 mm, which indicates that  $m_w$  is 490.9 g per meter. In contrast, the physical tail, which has a length of 8 cm, weighs 0.54 g, resulting in a mass of 6.75 g per meter, which is 1.38% of the virtual mass.

# 4.3 Dynamic Modeling of a Tail Using Euler-Bernoulli Beam Theory

The application of Lighthill's theory requires knowing the shape trajectory of the base-actuated flexible tail. We propose a multi-segment approach to the modeling of the tail that undergoes large deformations in Section 4.4. But in this section we first develop a comparative approach based on linear beam theory, which has been widely used in the relevant literature. We focus on
beam modeling with the robot anchored, as typically adopted in the literature [19, 122]. Fig. 4.2 illustrates this approach; the dashed line represents a rotating frame, which coincides with the line tangential to the tail at the base, and the blue solid curve represents the shape of the flexible caudal fin. The transverse displacement of points on the flexible tail relative to the rotating base, due to the beam's vibration, is given by  $w(\tau, t)$ . We note that the tail is assumed to be under small deformation in accordance with the Euler-Bernoulli theory. In other words, the motion direction of each point along the flexible tail is perpendicular to the dashed line.



Figure 4.2: Passive flexible tail modeled by Euler-Bernoulli linear beam approach.

The tail is excited by the oscillating support at the point *A*, with:

$$\left[\frac{\partial w_1(\tau,t)}{\partial \tau}\right]_{\tau=0} = \left[\frac{\partial w_2(\tau,t)}{\partial \tau}\right]_{\tau=0} = \alpha(t).$$
(4.1)

Here  $w_1(\tau, t)$  denotes the beam displacement in the inertial frame and  $w_2(\tau, t)$  accounts for the rigid-body (rotating base) motion:

$$w_1(\tau, t) = w_2(\tau, t) + w(\tau, t),$$
  
 $w_2(\tau, t) = \tau \cdot \alpha(t).$  (4.2)

The forced underwater vibration of the flexible beam is described by the following equation [122, 130]

$$\frac{\partial^2}{\partial \tau^2} \left[ EI(\tau) \frac{\partial^2 w_1(\tau, t)}{\partial \tau^2} \right] + m(\tau) \frac{\partial^2 w_1(\tau, t)}{\partial t^2} = p(\tau, t), \tag{4.3}$$

where *E* denotes the Young's modulus of the flexible tail, *I* denotes the area moment of inertia,  $m(\tau)$  denotes the mass of the tail per unit length and  $p(\tau,t)$  denotes the transverse loading on the tail which is caused by the interactions between the tail and the surrounding aquatic environment that can be evaluated by Lighthill's theory. Unlike [19, 122], in this work we also consider distributed viscous damping introduced by the internal resistance opposing the strain velocity [130], which leads to

$$\frac{\partial^2}{\partial \tau^2} \left[ EI(\tau) \left( \frac{\partial^2 w_1(\tau,t)}{\partial \tau^2} + \kappa \frac{\partial^3 w_1(\tau,t)}{\partial \tau^2 \partial t} \right) \right] + m(\tau) \frac{\partial^2 w_1(\tau,t)}{\partial t^2} = p(\tau,t),$$
(4.4)

where  $\kappa$  is the stiffness proportionality factor for Rayleigh damping.

Similar to [14, 15], we assume that the tail itself has negligible mass compared to the added mass effects. Then following Lighthill's theory, we obtain:

$$EI\left[\frac{\partial^4 w(\tau,t)}{\partial \tau^4} + \kappa \frac{\partial^5 w(\tau,t)}{\partial \tau^4 \partial t}\right] + m_w \frac{\partial^2 w(\tau,t)}{\partial t^2} = P_{\text{eff}}(\tau,t)$$
(4.5)

in which

$$\begin{split} P_{\rm eff}(\tau,t) = & -EI \bigg[ \frac{\partial^4 w_2(\tau,t)}{\partial \tau^4} + \kappa \frac{\partial^5 w_2(\tau,t)}{\partial \tau^4 \partial t} \bigg] \\ & -m_w \frac{\partial^2 w_2(\tau,t)}{\partial t^2} \end{split}$$

represents the effective distributed dynamic loading caused by the prescribed support excitations, and  $m_w$  is virtual mass per unit length defined previously, accounting for the added mass effect. Since the first two terms of  $P_{\text{eff}}(\tau, t)$  are equal to zero as  $w_2(\tau, t) = \tau \cdot \alpha(t)$ , the tail beam can be modeled as

$$EI\left[\frac{\partial^4 w(\tau,t)}{\partial \tau^4} + \kappa \frac{\partial^5 w(\tau,t)}{\partial \tau^4 \partial t}\right] + m_w \frac{\partial^2 w(\tau,t)}{\partial t^2} = -m_w \tau \cdot \ddot{\alpha}(t).$$
(4.6)

Using the modal analysis method, we can express the solution of (4.6) as the sum of an infinite number of modes:

$$w(\tau,t) = \sum_{i=1}^{\infty} \phi_i(\tau) \eta_i(t), \qquad (4.7)$$

where  $\phi_i(\tau)$  is the beam shape for the *i*th mode and  $\eta_i(t)$  is the corresponding generalized coordinate. There is no concentrated force in the  $\hat{n}$  direction with the anchored body assumption, which implies that the boundary conditions for  $w(\tau, t)$  is the same as those for a cantilever beam:

$$w(0,t) = 0, \qquad \frac{\partial w}{\partial \tau}(0,t) = 0,$$
$$EI\frac{\partial^2 w}{\partial \tau^2}(L,t) = 0, \qquad EI\frac{\partial^3 w}{\partial \tau^3}(L,t) = 0.$$
(4.8)

The mode shape  $\phi_i(\tau)$  takes the form

$$\phi_i(\tau) = (\cos\beta_i\tau - \cosh\beta_i\tau) - \delta_i(\sin\beta_i\tau - \sinh\beta_i\tau), \qquad (4.9)$$

where  $\beta_i$  can be obtained by solving

$$1+\cos\beta_i L\cosh\beta_i L=0,$$

and

$$\delta_i = rac{\coseta_i L + \cosheta_i L}{\sineta_i L + \sinheta_i L}.$$

With the damping ratio for the *i*th mode

$$\xi_i = \frac{\kappa \omega_i}{2},$$

where  $\omega_i$  is the natural frequency for the *i*th mode,

$$\omega_i = \beta_i^2 \sqrt{\frac{EI}{m_w}},\tag{4.10}$$

 $\eta_i(t)$  can be solved from

$$\frac{d^2 \eta_i(t)}{dt^2} + 2\xi_i \omega_i \frac{d\eta_i(t)}{dt} + \omega_i^2 \eta_i(t) = \frac{Q_i(t)}{M_i}, \quad i = 1, 2...$$
(4.11)

where

$$Q_i(t) = \int_0^L \phi_i(\tau) P_{\text{eff}}(\tau, t) \mathrm{d}\tau, \qquad (4.12)$$

and

$$M_i = \int_0^L \phi_i(\tau)^2 m_w \mathrm{d}\tau. \tag{4.13}$$

When the tail shaft oscillates sinusoidally, we can obtain a closed-form solution for  $\eta_i(t)$ . Consider in particular the following form for the base angle

$$\alpha(t) = \alpha_0 + \alpha_A \sin(\omega_\alpha t + \phi_\alpha), \qquad (4.14)$$

where  $\alpha_0$ ,  $\alpha_A$ ,  $\omega_\alpha$ , and  $\phi_\alpha$  denote the bias, amplitude, frequency, and initial phase of the tail beat, respectively. Then we have

$$\eta_i(t) = \frac{m_w \alpha_A \omega_\alpha^2 \int_0^L \tau \phi_i(\tau) d\tau}{M_i Z_i \omega_\alpha} \sin(\omega_\alpha t + \phi_\alpha + \phi_i), \qquad (4.15)$$

where

$$Z_i = \sqrt{(2\omega_i\xi_i)^2 + \frac{1}{\omega_\alpha^2}(\omega_\alpha^2 - \omega_i^2)^2}$$

is the magnitude of the impedance function and

$$\phi_i = \arctan\left(\frac{2\omega_{\alpha}\omega_i\xi_i}{\omega_{\alpha}^2 - \omega_i^2}\right)$$

is the phase lag of the oscillation relative to  $\alpha(t)$ . The total hydrodynamic force acting on the tail is then

$$\vec{F} = \int_{0}^{L} \vec{f}(\tau) d\tau - \left[\frac{1}{2}m_{w}v_{\perp}^{2}\hat{m}\right]_{\tau=L},$$
(4.16)

where  $\vec{f}(\tau) = -m_w \left[ (\tau \ddot{\alpha} + \frac{\partial^2 w}{\partial t^2}) \hat{n} - \dot{\alpha} (\tau \dot{\alpha} + \frac{\partial w}{\partial t}) \hat{m} \right]$ . The corresponding moment relative to the

center C of the robot body is

$$\vec{M} = \int_0^L \vec{r}_{C\tau} \times \vec{f}(\tau) d\tau + \vec{r}_{CL} \times \left( -\left[\frac{1}{2}m_w v_\perp^2 \hat{m}\right]_{\tau=L} \right), \tag{4.17}$$

where  $\vec{r}_{C\tau}$  and  $\vec{r}_{CL}$  denote the vectors from the body center C to the point  $\tau$  and L on the tail, respectively.

# 4.4 Dynamic Modeling of a Tail Using Multi-segment Approximation

A fundamental underlying assumption in establishing the model described in the previous section is that the flexible tail is under small deformation. However, based upon observation of the dynamic shape of the tail, this assumption does not always hold, especially when the tail undergoes large angular displacement and/or high-frequency oscillations. In this section, we propose a novel model by representing the tail as multiple rigid elements connected in series through torsional springs and dampers, to capture the large deformation of the beam. *N* rigid segments, with equal length of *l*, are used to represent the tail, as illustrated in Fig. 4.3. Each segment is linked with its neighboring segments through joints modeled by a torsional spring  $K_S$  and a viscous damper  $K_D$ .

Following [125], we can evaluate the stiffness of each torsional spring as

$$K_S = \frac{Edh^3}{12l},\tag{4.18}$$

with *h* denoting the thickness of tail.  $K_D$  can be evaluated as  $K_D = \kappa K_S$ , where  $\kappa$  is the proportional constant as defined in the previous section. Following Lighthill's large-amplitude elongated-body



Figure 4.3: Passive flexible tail modeled by multiple rigid segments.

theory, we need to compute the motion of every point along the tail over time, in order to evaluate the tail actuation-induced hydrodynamic force. Consequently, we need to know the joint angles  $\alpha_i$ , made by the *i*th link with respect to the negative *x*-axis (as illustrated in Fig. 4.3), so that (3.8) and (3.9) can be applied to evaluate the hydrodynamic forces on the tail section. The displacement  $\vec{r}_{\tau_i}$  of every point  $\tau_i$  on the *i*th segment in the inertial frame can be described as:

$$\vec{r}_{\tau_i} = l \cdot \left[\sum_{k=1}^{i-1} \hat{m}_k\right] + \tau_i \hat{m}_i, \qquad (4.19)$$

then the velocity perpendicular to the *i*th segment is

$$v_{\tau_i\perp} = l \cdot \left[\sum_{k=1}^{i-1} \dot{\alpha}_k \cos(\alpha_k - \alpha_i)\right] + \tau_i \dot{\alpha}_i.$$
(4.20)

Therefore, the force density acting on the *i*th segment can be evaluated as

$$\vec{f}_{\tau_{i}} = -m_{w} \frac{d}{dt} (v_{\tau_{i}} \perp \hat{n}_{i})$$

$$= -m_{w} \left\{ l \sum_{k=1}^{i-1} \left[ \ddot{\alpha}_{k} \cos(\alpha_{i} - \alpha_{k}) - \dot{\alpha}_{k} \sin(\alpha_{i} - \alpha_{k}) (\dot{\alpha}_{i} - \dot{\alpha}_{k}) \right] + \tau_{i} \ddot{\alpha}_{i} \right\} \hat{n}_{i}$$

$$+ m_{w} \dot{\alpha}_{i} \left\{ l \sum_{k=1}^{i-1} \left[ \dot{\alpha}_{k} \cos(\alpha_{i} - \alpha_{k}) \right] + \dot{\alpha}_{i} \tau_{i} \right\} \hat{m}_{i}.$$

$$(4.21)$$

The total reactive force  $\vec{F}_i$  on the *i*th segment, and the moment  $\vec{M}_i$  relative to point  $A_{i-1}$  can be evaluated using

$$\vec{F}_i = \int^l \vec{f}_{\tau_i} d\tau, \quad \vec{M}_i = \int^l \tau_i \hat{m}_i \times \vec{f}_{\tau_i} d\tau.$$
(4.22)

Defining  $\vec{F}_{Ai}$  and  $\vec{M}_{Ai}$  to be the force and the moment exerted by the (i+1)th segment on the *i*th segment, respectively, we can express the interactions between adjacent segments as,

$$\vec{F}_{A_i} = \vec{F}_{i+1} + \vec{F}_{A_{i+1}}, \tag{4.23}$$

$$\vec{M}_{A_{i}} = \vec{M}_{i+1} + \vec{M}_{A_{i+1}} + l\hat{m}_{i} \times \vec{F}_{A_{i+1}}.$$
(4.24)

For the last segment of the tail, as illustrated in Fig. 4.3, the reactive force and moment are

$$\vec{F}_{A_N} = -\frac{1}{2} m_w \left[ l \sum_{k=1}^N \dot{\alpha}_k \cos(\alpha_N - \alpha_k) \right]^2 \hat{m}_N + m_w \left[ l \sum_{k=1}^N \dot{\alpha}_k \cos(\alpha_N - \alpha_k) \right] \cdot \\ \left[ l \sum_{k=1}^N \dot{\alpha}_k \sin(\alpha_N - \alpha_k) \right] \hat{n}_N$$

$$(4.25)$$

$$\vec{M}_{A_N} = 0.$$
 (4.26)

Defining  $\vec{M}_{(S+D)_i}$  the moment produced by the spring and damper at joint  $A_i$ , then the moment balance equation implies, for  $i = 1, 2, \dots, N-1$ ,

$$\vec{M}_{A_i} = \vec{M}_{(S+D)_i},\tag{4.27}$$

where

$$\vec{M}_{(S+D)_{i}} = \left[ K_{S}(\alpha_{i+1} - \alpha_{i}) + K_{D}(\dot{\alpha}_{i+1} - \dot{\alpha}_{i}) \right] \hat{z}.$$
(4.28)

Eq. (4.27) has (N-1) scalar equations involving (N-1) unknown variables  $\alpha_2, \dots, \alpha_N$ , which is solvable.

 $\vec{F}_{A_0}$  is the force that the flexible tail exerts on the robotic fish body, which can be written as  $\vec{F}_{A_0} = F_{hx} \cdot \hat{x} + F_{hy} \cdot \hat{y}$ , where  $F_{hx}$  and  $F_{hy}$  are the components of  $F_A$  along  $\hat{x}$  and  $\hat{y}$ , respectively. The moment to the center of the body caused by the oscillation of the tail can be evaluated as

$$\vec{M}_C = \vec{M}_{A_0} - c\hat{x} \times \vec{F}_{A_0}.$$
(4.29)

It is clear that  $\vec{M}_C$  is along the *z* direction, which we denote as  $M_{hz}$ .

### 4.5 Experimental Model Validation

To evaluate the dynamic models described in Sections 4.3 and 4.4, we have conducted experiments with the free-swimming robotic fish prototype shown in Fig. 4.4. The robot has a simple mechanism for actuation, and it satisfies most of the assumptions used in the modeling work, which facilitates model validation. In particular, the height of the tail does not change abruptly along the length direction, thus meeting the "elongated body" requirement. The thickness of the tail is

much less than its height, so that the actual mass of the tail is much less than the virtual mass as assumed. Through a chain transmission mechanism, a servomotor (HS-5085MG from Hitec) is able to control the angular position of the tail shaft and thus the tail deflection angle  $\alpha$ , i.e. sinusoidal motion, accurately. On the other hand, in the modeling work, we assume that the robot is anchored, as typically adopted in the literature. This assumption, of course, does not hold fully during free-swimming experiments, which might explain the slight discrepancies between the model predictions and experimental measurement in those experiments. The tail was a rectangular plastic slice, which was 8 cm long, 2.5 cm wide (deep), and 0.3 mm thick.



Figure 4.4: A free-swimming robotic fish prototype used for model validation.

#### 4.5.1 Parameter identif cation

The same experimental prototype (with a different tail) was used in [14] to validate a dynamic model for a robotic fish with a rigid tail. The following parameters for the robot were identified in [14]: c = 0.07 m,  $m_b = 0.311$  kg,  $\rho = 1000$  kg/m<sup>3</sup>, S = 0.0108 m<sup>2</sup>,  $J_{bz} = m_b(2c)^2/12$ . The added masses and inertias are calculated by approximating the robot body as a prolate spheroid [12, 112]:

$$-X_{\dot{V}_{Cx}} = 0.0621 \text{ kg}, -Y_{\dot{V}_{Cy}} = 0.2299 \text{ kg}, \text{ and } -N_{\dot{\omega}_z} = 1.0413 \times 10^{-4} \text{ kg} \cdot \text{m}^2.$$

The drag and lift coefficients,  $C_D$ ,  $C_L$ , and  $C_M$  are also identified empirically for the robotic fish with a rigid tail and then used in validating the flexible tail models. In particular, we have tuned these parameters to match the turning radius, turning period, and the average of  $|\beta|$  obtained in simulation, with the experimental measurements recorded for a particular tail beat pattern. We set amplitude  $\alpha_A = 13.6^\circ$ , frequency  $\omega_{\alpha} = 1.8\pi$  rad/s (0.9 Hz) and evaluate the suitable drag and lift coefficients for the tail-beat bias  $\alpha_0$  equal to 20°, 30° and 40° as [15]. Using least-square-error fitting, we obtain  $C_D = 0.276$ ,  $C_L = 4.5$ , and  $C_M = 7.4 \times 10^{-4}$  kg·m<sup>2</sup>, when  $\alpha_0 = 0$ .

For measuring the Young's modulus, we set up the experiment shown in Fig. 4.5, and evaluate *E* using:

$$E = \frac{L_b^3 P_L}{3I_b w_L},\tag{4.30}$$

where  $L_b$  and  $I_b$  are the length and the area of moment inertia of the testing beam, respectively.  $P_L$  is the load at the tip end, and  $w_L$  is the end's displacement. A plastic beam (the same material as the flexible tail) is clamped at the upper surface of a rectangular block, which is fixed on a piston that can move up and down along the stand. A custom LabVIEW (2011 SP1) virtual instrument graphical user interface (GUI) is developed to perform the data acquisition through a dSPACE system (RTI 1104, dSPACE). The force exerted on the beam is captured by a load cell (GS-10, Transducer Techniques) with a custom-made amplifier circuit. The displacement of the beam tip is measured with a laser sensor (OADM 20I6441/S14F, Baumer Electric). Prior to experiment, the system is calibrated with a weight applied to the load cell. Three sets of data are collected to calculate the Young's modulus, and in each set, a least square error method is adopted to approximate the slope between the force and displacement. Fig. 4.6 shows results for three tests, indicating that  $E \in [1.41, 1.51]$  GPa. In this work, we takes the average E = 1.48 GPa.



Figure 4.5: Experimental setup for measuring the Young's modulus of the flexible tail.

When modeling a tail using the multi-segment approximation, the number of rigid segments affects both modeling accuracy and computational complexity [127]. Specifically, a higher number of elements results in a more accurate model, but is also more computation-intensive. Fig. 4.7 shows the simulated responses of the beam tip displacement (relative to the x axis along the body) to a sinusoidal base excitation, when different numbers of segments are used to model the flexible beam. The properties of the beam used in the simulation are same as those of the beam identified



Figure 4.6: Experimental results for measuring the Young's modulus of the flexible tail.

from the experiments. It can be observed that, the beam tip displacements gradually converge to each other when the number of segments increases. Fig. 4.8 shows the running time needed for the simulation illustrated in Fig. 4.7, which was conducted with Matlab/Simulink on a desktop PC (Dell Vostro 460 with 3.1 GHz Intel i5-2400 CPU and 4 GB memory). In particular, for all cases, the fundamental sampling time in Simulink is fixed at 0.000167 s, the simulation time is set to be 10 s in total, and the elapsed CPU time is obtained using the Matlab macro *cputime*. We can see that the computation time increases rapidly with the number of rigid segments used in the simulation. A five-segment approximation achieves a sound tradeoff between the modeling accuracy and computational efficiency, and therefore is adopted in this study.

According to Eq. (4.18), we have  $K_S = 5.2 \times 10^{-3}$  N·m that is used in the model using multisegment approximation. The values of  $\kappa$  are identified empirically by fitting the data of forward



Figure 4.7: Displacement of the tail tip generated by the model using different number of rigid segments.

speed versus beating frequency for both models. In particular, we choose the value of  $\kappa$  such that the simulated steady-state speed of the robot, when  $\alpha_0 = 0^\circ$ , matches the experimental results when the tail beats at 0.45 Hz, 0.9 Hz and 1.35 Hz, respectively (for which cases the flexible tail undergoes relatively small deformations). This results in  $\kappa = 0.0846$  s for the Euler-Bernoulli beam model and  $\kappa = 0.0731$  s for the multi-segment approximation model. We take the average  $\kappa = 0.079$  s, which leads to  $K_D = 4.1 \times 10^{-3}$  N·m·s, using  $K_D = \kappa K_S$ . These parameters are then applied in the simulation of all other cases.

#### 4.5.2 Model verif cation

Fig. 4.9 compares the two models and experimental measurements in terms of the steady-state speed of the robotic fish with respect to the actuation frequency. The trends are similar for the two



Figure 4.8: Computation time needed to simulate the model using different numbers of rigid segments.

model predictions. In particular, the robot's speed increases with the frequency up to a threshold value (1.35 Hz in the simulated case) and then starts to drop. While the predictions from both models match the experiments relatively well (due to the tuning process for parameter  $\kappa$  as described in the previous subsection), the discrepancy between the predictions becomes larger as the actuation frequency increases. The flexible tail is subjected to the added mass effect as explained in Eq. (3.8). Under a high frequency excitation, the lateral hydrodynamic loading on the tail increases and causes larger deformation of the tail. By using linear Euler-Bernoulli beam theory, we assume that the tail is under small deformation, and thus the motion direction of each point along the flexible tail is perpendicular to the dashed line in Fig. 4.2. The latter assumption no longer holds when the excitation frequency gets high due to the large deformation, which explains the poor prediction performance of the linear beam model at relatively high frequencies. On the other

hand, the model using the multi-segment approximation matches the experimental measurement closely throughout the actuation frequency range used in the experiments.



Figure 4.9: Comparison between model predictions and experimental measurement of the speed versus tail-beat frequency. The amplitude is fixed at  $13.6^{\circ}$ .

To further compare the two models under different conditions, a second experiment has been conducted with another tail of different dimensions, which is 2.34 cm high and 9.8 cm long. Following Eq. (4.18) and the linear relationship  $K_D = \kappa K_S$ , the values of  $K_S$  and  $K_D$  are updated from the original  $K_S = 5.2 \times 10^{-3}$  N·m and  $K_D = 4.1 \times 10^{-3}$  N·m·s to  $K_S = 4.0 \times 10^{-3}$  N·m and  $K_D = 3.13 \times 10^{-3}$  N·m·s, respectively. Fig. 4.10 shows the comparison between the two models and the experimental measurements for the robot with the new tail. Consistent with the results in Fig. 4.9, one can see that the model using the multi-segment approximation matches the experimental measurement closely for all actuation frequencies, while the linear beam model performs well only for low frequency actuation.



Figure 4.10: Comparison between model predictions and experimental measurement of the speed versus tail-beat frequency, with the new tail. The amplitude is fixed at  $13.6^{\circ}$ .

To further assess the proposed multi-segment model, we conduct an experiment involving transients in forward motion, where the robot started from at rest. Fig. 4.11(a) and (b) compare the model predictions of the X/Y-coordinate time-trajectories of the robot with the experimental measurements, and Fig. 4.11(c) compares the predicted and actual robot paths in the XY-plane. It can be seen that the proposed model is able to capture well both the transient and steady-state behaviors of the robot.

We have conducted additional experiments to compare the time-dependent shape of the flexible tail with those predicted by the models. As can be seen in Fig. 4.12, the free swimming robot shown in Fig. 4.4 is fixed by a bracket and set to oscillate the tail. A Casio Exilim EX-FH25 high-speed camera with a frame rate of 120 frames/s is used to record the tail's motion from above.

Figs. 4.13 and 4.14 compare the measured time-dependent tail shape and those predicted by

the two models for 0.4 Hz and 0.9 Hz, respectively. To save space, we show every ninth frame for half a period of the tail oscillation. It can be seen that when the tail beats at the relative low frequency, 0.4 Hz, both models produce very close approximations of the tail shape. However, at 0.9 Hz, when the beam deformation is bigger, the multi-segment modeling approach produces much more precise predictions of the beam shape than the linear beam theory-based approach.

# 4.6 Conclusions

In this chapter, we have developed a model for robotic fish with a base-actuated flexible tail. The tail is modeled as multiple rigid segments connected by springs and dampers, and Lighthill's elongated-body theory is used to evaluate the tail-generated hydrodynamic forces. For comparison, we have also constructed a model using linear beam theory. We compare predictions of both models to experimental results with a robotic fish, in terms of steady-state cruising speeds and dynamic tail shapes under different actuation frequencies. From these results, we conclude that when the tail is excited under a relatively low frequency, and consequently experiences small deformation, both models produce similar predictions that are close to experimental measurements. However, when the actuation frequency increases, the two models differ, and the model using multi-segment approximation is able to predict much better the robot speed and the tail shape. Additional experimental results also indicate that the proposed model is capable of capturing the transient dynamics of the robot.

In summary, the work presented in this chapter provides a computationally efficient and accurate model for capturing large tail deformation and the resulting hydrodynamic force for a tailactuated robotic fish. The model will facilitate effective tail design optimization and controller development for such robots. While one could also use nonlinear beam models and computational fluid dynamics (CFD) to achieve faithful modeling of the tail, the latter approach would be much more computationally expensive, and would be difficult to integrate with the robot dynamics for controller design.

This work can be extended in several directions. First, although this chapter has focused on the case of a rectangular tail for ease of presentation, the mathematical derivation itself is general and the approach can be extended to tails of other shapes (e.g., trapezoidal) following a similar treatment, where the properties of each segment (mass, inertia) and each joint (spring and damper constants) will depend on the local shape. Second, we are particularly interested in using the proposed model to understand the effect of tail shape and stiffness properties on the robot's locomotion performance, and exploit such understanding for design optimization. Finally, we will utilize the proposed model to design controllers for flexible tail-actuated robotic fish.



Figure 4.11: Comparison between multi-segment model predictions and experimental measurement for forward swimming (including transients): (a) time trajectory of *X*-coordinate of the robot; (b) time trajectory of the *Y*-coordinate of the robot; (c) path of the robot in the *XY*-plane. For both the experiment and simulations, the amplitude and frequency of the tail beat are fixed at  $13.6^{\circ}$  and 0.9 Hz, respectively.



Figure 4.12: Experimental setup to capture the dynamic shape of a flexible tail actuated at the base.



Figure 4.13: Comparison between experimental measurement of the time-dependent tail shape with model predictions. The tail beats at 0.4 Hz with  $0^{\circ}$  bias and  $14^{\circ}$  amplitude. The black solid line, blue dashed line with circles and the red dash-dot line imply the experimental measurement, predictions from multi-segment model and Euler-Bernoulli beam model, respectively.



Figure 4.14: Comparison between experimental measurement of the time-dependent tail shape with model predictions. The tail beats at 0.9 Hz with 0° bias and 13.6° amplitude. The black solid line, blue dashed line with circles and the red dash-dot line imply the experimental measurement, predictions from multi-segment model and Euler-Bernoulli beam model, respectively.

# Chapter 5

# **Control-oriented Averaging of Tail-actuated Robotic Fish Dynamics**

# 5.1 Introduction

Much of the work in the area of robotic fish has focused on the design ( [5, 6, 8, 10, 131]) and dynamic modeling [11, 14, 15, 45, 92, 95, 96, 132]. Due to the rhythmic nature of body and fin movements of robotic fish, averaging represents a promising approach in obtaining models that simplify motion analysis and control design [133]. While classical averaging [134], which simply integrates the original vector field over one period, is easy to implement, it cannot be applied directly to robotic fish dynamics, because the latter typically involves large-amplitude oscillatory inputs and fails to satisfy the slow-dynamics assumption required by classical averaging theory. Geometric averaging [135, 136], on the other hand, can handle systems with highly oscillatory inputs, but as we show in this chapter, for an original dynamic model that is reasonably accurate (and complex), the resulting average model is overly complicated and has limited use for system analysis and control design.

In this chapter, a novel averaging approach is proposed for robotic fish dynamics, with a focus on planar motions of a rigid tail-actuated robotic fish. Note that among various actuation mechanisms, actuation with an oscillating caudal fin is particularly attractive (e.g., [6,11,12,44,45]). This is because the latter is easy to realize, enables both forward swimming and turning maneuvers, and

leaves the majority of the robot body free of moving parts. We adopt a model where Lighthill' large-amplitude elongated-body theory [25] is used to evaluate the tail-generated hydrodynamic force, since such a model strikes a sound balance between fidelity and simplicity [15].

The proposed averaging approach consists of properly scaling the force and moment terms in the dynamic model and then conducting classical averaging. To identify the scaling functions, a large number of tail-beat patterns are used in simulation of the original dynamics; for each tail-beat pattern (bias, amplitude, and frequency), the scaling coefficients for the force and moment terms are determined such that the corresponding average model will result in turning radius and turning period at the steady state that best match those of the original dynamics. It is found that, for a given set of parameters for the robotic fish model, its force scaling function is a constant (thus independent of the tail-beat pattern) while its moment scaling function is linear in the tail-beat bias but independent of tail-beat frequency and amplitude.

The proposed averaging model is evaluated extensively with simulation and experimental results, including the comparison of predictions of turning period/radius and cruising speed at the steady state, by the original model and the average model, respectively. Also included is the validation of the average models ability to predict transient behavior of the robot. Most of the simulation and experimental scenarios used in the evaluation have not been used in identifying the scaling coefficients, which provides strong support for the effectiveness of the average model.

To gain analytical insight into the obtained average model, we analyze the existence and stability of its equilibrium points. In particular, with several reasonable assumptions, we establish the existence and uniqueness of equilibrium points for the (approximate) average model under a given tail-beat pattern. Furthermore, through linearization analysis, we show that any equilibrium of the average model is locally asymptotically stable.

Finally, to illustrate the utility of the proposed average model in motion analysis, we present

a semi-analytical framework for modeling the steady turning of robotic fish. Based upon the proposed average model, the force and moment balance equations during steady turning are set up explicitly in terms of three unknowns, the turning radius, angular speed (equivalent to turning period information) and the angle of attack. These *algebraic* equations can then be solved to get the turning-related parameters for any given tail-beat pattern. Such a framework is different from existing work (e.g. [14,15,137,138]), where dynamical equations are used for simulation and turning parameters can only be found by observing the simulated trajectories. We note that the average model also holds potential for controller design; for example, Chen *et al.* has used a preliminary form of this model in backstepping-based control design for a robotic fish [139].

The remainder of the chapter is organized as follows. In Section 5.2, we summarize the dynamic model for a tail-actuated carangiform robotic fish. In Section 5.3 we review the first-order geometric averaging method and the classical averaging method, and discuss why they are inadequate for averaging the robotic fish dynamics. The proposed averaging method is developed and evaluated in Section 5.4. Analysis of the resulting average model is presented in Section 5.5. The semi-analytical framework for turning is described in Section 5.6. A nonlinear controller design for the robotic fish to achieve trajectory tracking using the average model is presented in Section 5.7. Finally, concluding remarks are provided in Section 5.8.

# 5.2 Dynamic Model of Robotic Fish

As described in Section 3.2 of Chapter 3, the equation of planar motion for a rigid body in an inviscid fluid is governed by the Kirchhoff's equations

$$(m_b - X_{\dot{V}_{Cx}})\dot{V}_{Cx} = (m_b - Y_{\dot{V}_{Cy}})V_{Cy}\omega_z + F_x,$$
(5.1)

$$(m_b - Y_{\dot{V}_{Cy}})\dot{V}_{Cy} = -(m_b - X_{\dot{V}_{Cx}})V_{Cx}\omega_z + F_y,$$
(5.2)

$$(J_{bz} - N_{\dot{\omega}_z})\dot{\omega}_z = (Y_{\dot{V}_{Cy}} - X_{\dot{V}_{Cx}})V_{Cx}V_{Cy} + M_z,$$
(5.3)

where  $F_x$ ,  $F_y$ ,  $M_z$  denote the external forces and moments about the body center, which are caused by the tail motion and the interaction between the body itself and the surrounding fluid, and can be evaluated by

$$\begin{cases} F_x = \langle (\int_0^L \vec{f}(\zeta) \mathrm{d}\zeta + \vec{F}_L), \hat{\mathbf{x}} \rangle - F_D \cos\beta + F_L \sin\beta, \\ F_y = \langle (\int_0^L \vec{f}(\zeta) \mathrm{d}\zeta + \vec{F}_L), \hat{\mathbf{y}} \rangle - F_D \sin\beta - F_L \cos\beta, \\ M_z = \int_0^L \vec{r}_{C\zeta} \times \vec{f}(\zeta) \mathrm{d}\zeta + \vec{r}_{CL} \times \vec{F}_L + M_D, \end{cases}$$
(5.4)

where  $\vec{r}_{C\zeta}$  denotes the vector from the body center to the point  $\zeta$  on the tail, i.e.,  $\vec{r}_{C\zeta} = -(c + \zeta \cos \alpha)\hat{x} - (\zeta \sin \alpha)\hat{y}$ , and *c* is the distance from the body center to the joint. "×" represents the cross product of vectors.

Finally, the dynamic model for the robotic fish can be summarized as

$$\begin{cases} \dot{V}_{Cx} = f_1(V_{Cx}, V_{Cy}, \omega_z) + f_4(t), \\ \dot{V}_{Cy} = f_2(V_{Cx}, V_{Cy}, \omega_z) + f_5(t), \\ \dot{\omega}_z = f_3(\omega_z) + f_6(t), \end{cases}$$
(5.5)

where

$$\begin{cases} f_1(V_{Cx}, V_{Cy}, \omega_z) = \frac{m_2}{m_1} V_{Cy} \omega_z - \frac{c_1}{m_1} V_{Cx} \sqrt{V_{Cx}^2 + V_{Cy}^2} \\ + \frac{c_2}{m_1} V_{Cy} \sqrt{V_{Cx}^2 + V_{Cy}^2} \arctan(\frac{V_{Cy}}{V_{Cx}}), \\ f_2(V_{Cx}, V_{Cy}, \omega_z) = -\frac{m_1}{m_2} V_{Cx} \omega_z - \frac{c_1}{m_2} V_{Cy} \sqrt{V_{Cx}^2 + V_{Cy}^2} \\ - \frac{c_2}{m_2} V_{Cx} \sqrt{V_{Cx}^2 + V_{Cy}^2} \arctan(\frac{V_{Cy}}{V_{Cx}}), \\ f_3(\omega_z) = (m_1 - m_2) V_{Cx} V_{Cy} - c_4 \omega_z^2 \operatorname{sgn}(\omega_z), \\ f_4(t) = -\frac{c_3}{m_1} \ddot{\alpha} \sin \alpha, \\ f_5(t) = +\frac{c_3}{m_2} \ddot{\alpha} \cos \alpha, \\ f_6(t) = -c_5 \ddot{\alpha} \cos \alpha - c_6 \ddot{\alpha}, \end{cases}$$

with  $c_1 = 1/2\rho SC_D$ ,  $c_2 = 1/2\rho SC_L$ ,  $c_3 = 1/2mL^2$ ,  $c_4 = 1/(J_3)K_D$ ,  $c_5 = 1/(2J_3)L^2mc$  and  $c_6 = 1/(3J_3)L^3m$ ,  $m_1 = m_b - X_{\dot{V}_{Cx}}$ ,  $m_2 = m_b - Y_{\dot{V}_{Cy}}$ ,  $J_3 = J_{bz} - N_{\dot{\omega}_z}$ , where *L* is the tail length and  $\ddot{\alpha}$  denotes the second-derivative of the tail angle  $\alpha$  with respect to time.

# 5.3 Averaging with Existing Methods

Since the tail-actuation for robotic fish is typically periodic, averaging is a useful tool for gaining insight into the effect of the input parameters (such as tail beat bias, period, and frequency) on the dynamics [11, 133], and for designing the controller [11, 139]. In this section we review some fundamental results on averaging theory [134, 136], apply them to the robotic fish model, and discuss their limitations.

#### 5.3.1 First-order Averaging for Systems with Oscillatory Inputs

To address the averaging of the dynamics for a robotic fish, one can consider first-order averaging for systems with oscillatory control input [136]. In particular, consider the following dynamics

$$\dot{x} = f(x) + (1/\varepsilon)g(x, t/\varepsilon), \tag{5.6}$$

where  $\varepsilon > 0$  is a small number, and  $[(1/\varepsilon)g(x,t/\varepsilon)]$  is a *T*-periodic, zero mean function in  $t/\varepsilon$  to indicate a high-amplitude and large-frequency oscillatory input. With the transformation of time,  $t/\varepsilon \mapsto \tau$ , one can obtain

$$\frac{dx}{d\tau} = \varepsilon f(x) + g(x,\tau).$$
(5.7)

The term  $\varepsilon f(x)$  can be treated as a perturbation to the primary vector field  $g(x, \tau)$ . Define the following

$$F(y,\tau) = \left( (\Phi_{0,\tau}^g)^* f \right)(y), \tag{5.8}$$

$$F_{av}(y) = \frac{1}{T} \int_0^T F(y,\tau) d\tau, \qquad (5.9)$$

where  $\Phi_{0,\tau}^g$  is the flow of the vector field g, and  $(\Phi_{0,\tau}^g)^* f$  is the pull-back of f by  $\Phi_{0,\tau}^g$ . According to the Variation of Constants formula, the solution  $x(\tau)$  is obtained by

$$x(\tau) = \Phi^{g}_{0,\tau}(y(\tau)),$$
 (5.10)

where  $y(\tau)$  is the solution to the system

$$dy/d\tau = \varepsilon F(y,\tau),\tag{5.11}$$

with  $y(0) = x_0$ . An approximation to the solution y can be obtained from the average equation

$$dz/d\tau = \varepsilon F_{av}(z). \tag{5.12}$$

It can be shown that [136]  $z(\tau) - y(\tau) = O(\varepsilon)$  on the time scale 1. Moreover, if the origin is an asymptotically stable equilibrium point for the average dynamics, then  $z(\tau) - y(\tau) = O(\varepsilon)$  as  $\varepsilon \to 0$  for all  $\tau \in \mathbb{R}_+$  and the differential equation (5.11) possesses a unique periodic orbit that is locally asymptotically stable and belongs to an  $O(\varepsilon)$ -neighborhood of the origin.

The pull-back used in Eq. (5.8) can be calculated as

$$(\Phi_{0,t}^{g})^{*}f$$

$$= f + \sum_{k=1}^{\infty} \int_{0}^{t} \cdots \int_{0}^{s_{k-1}} (\mathrm{ad}_{g(s_{k})} \cdots \mathrm{ad}_{g(s_{1})}f) ds_{k} \cdots ds_{1},$$
(5.13)

where  $ad_X Y = [X, Y]$  is the Lie bracket of *X* and *Y*, or the Lie derivative of *Y* with respect to *X*. The convergence of the infinite sum in (5.13) is generally not guaranteed unless the series contains only a finite number of non-vanishing terms.

#### 5.3.2 First-Order Averaging for Robotic Fish Model

In [133], Morgansen *et al.* applied the first and higher-order averaging method to a simplified carangiform robotic fish model and demonstrated its effectiveness with trajectory stabilization controller design. We now apply the first-order averaging analysis to the robotic fish model presented in Section 5.2.

We consider the following periodic pattern for the tail deflection angle:

$$\alpha(t) = \alpha_0 + \alpha_A \sin(\omega_\alpha t), \qquad (5.14)$$

where  $\alpha_0$ ,  $\alpha_A$  and  $\omega_{\alpha}$  denote the bias, amplitude, and frequency of the tail beat, respectively.

Define  $\varepsilon = 1/\omega_{\alpha}$  and  $\tau = t/\varepsilon$ , which allows us to express

$$\alpha(t) = \alpha_0 + \alpha_A \sin(\tau), \qquad (5.15)$$

$$\ddot{\alpha}(t) = -\frac{1}{\varepsilon}\omega_{\alpha}\alpha_{A}\sin(\tau).$$
(5.16)

Define the states  $x_1 = u$ ,  $x_2 = v$ ,  $x_3 = \omega$ . The system (5.5) can be rewritten in the form of (5.6)

$$\dot{x}(t) = f(x) + \frac{1}{\varepsilon}g(\frac{t}{\varepsilon}), \qquad (5.17)$$

where  $g(\cdot)$  is zero mean over a period of *T*. To facilitate the computation of the average, we first use the second-order Taylor series expansion to approximate  $\sin \alpha \approx \alpha - \frac{1}{6}\alpha^3$  and  $\cos \alpha \approx 1 - \frac{1}{2}\alpha^2$ . The terms like  $\arctan(V_{Cy}/V_{Cx})$  in Eq. (5.5) impede the averaging process due to its highly nonlinear nature. To address this challenge, we further simplify (5.5) with the assumptions that  $V_{Cx} \gg V_{Cy}$  and  $V_{Cx} > 0$ . In other words, the forward velocity is positive, and much larger than the sideway velocity. These assumptions are reasonable, and they imply that the terms  $\sqrt{V_{Cx}^2 + V_{Cy}^2}$  and  $\arctan(V_{Cy}/V_{Cx})$  can be approximated by  $V_{Cx}$  and  $V_{Cy}/V_{Cx}$ , respectively. We also neglect the coupling term  $(m_1 - m_2)V_{Cx}V_{Cy}$ , when evaluating the total moment in Eq. (10), based upon the observation that it is usually less than 2% of the moment  $M_z$  that is caused by external forces. With

the aforementioned simplifications, the system (5.5) can be re-written as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = f(x) + \frac{1}{\varepsilon}g(\frac{t}{\varepsilon}) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix}$$
(5.18)

where

$$f_{1}(x) = \frac{m_{2}}{m_{1}}x_{2}x_{3} - \frac{k_{1}}{m_{1}}x_{1}^{2} + \frac{k_{2}}{m_{1}}x_{2}^{2} + k_{4},$$

$$f_{2}(x) = -\frac{m_{1}}{m_{2}}x_{1}x_{3} - \frac{(k_{1}+k_{2})}{m_{2}}x_{1}x_{2} + k_{5},$$

$$f_{3}(x) = -k_{3}x_{3}^{2}\text{sgn}(x_{3}) - k_{6},$$

$$g_{1}(t) = \left\{k_{7}(k_{8}\sin\omega_{\alpha}t - k_{9}\cos2\omega_{\alpha}t - k_{10}\sin^{3}\omega_{\alpha}t + k_{11}\cos2\omega_{\alpha}t - k_{12}\cos4\omega_{\alpha}t)\right\},$$

$$g_{2}(t) = -k_{13}(k_{14}\sin\omega_{\alpha}t + k_{15}\cos2\omega_{\alpha}t - k_{16}\sin^{3}\omega_{\alpha}t),$$

$$g_{3}(t) = \left\{k_{17}(k_{18}\sin\omega_{\alpha}t + k_{19}\cos2\omega_{\alpha}t - k_{20}\sin^{3}\omega_{\alpha}t) + k_{21}\sin\omega_{\alpha}t\right\}.$$

with  $k_1 = \frac{1}{2}\rho SC_D$ ,  $k_2 = \frac{1}{2}\rho SC_L$ ,  $k_3 = \frac{K_D}{J_3}$ ,  $k_4 = \frac{m}{12m_1}L^2\alpha_A^2\omega_\alpha^2(3\alpha_A - \frac{3}{2}\alpha_0^2\alpha_A - \frac{3}{8}\alpha_A^3)$ ,  $k_5 = \frac{m}{4m_2}L^2\alpha_0\alpha_A^2\omega_\alpha^2$ ,  $k_6 = \frac{L^2}{4J_3}mc\alpha_0\alpha_A^2\omega_\alpha^2$ ,  $k_7 = \frac{m}{12m_1}L^2\alpha_A\omega_\alpha^2$ ,  $k_8 = 6\alpha_0 - \alpha_0^3$ ,  $k_9 = 3\alpha_A - \frac{3}{2}\alpha_0^2\alpha_A$ ,  $k_{10} = 3\alpha_0\alpha_A^2$ ,  $k_{11} = \frac{1}{2}\alpha_A^3$ ,  $k_{12} = \frac{1}{8}\alpha_A^3$ ,  $k_{13} = \frac{m}{4m_2}L^2\alpha_A\omega_\alpha^2$ ,  $k_{14} = 2 - \alpha_0^2$ ,  $k_{15} = \alpha_0\alpha_A$ ,  $k_{16} = \alpha_A^2$ ,  $k_{17} = \frac{L^2}{4J_3}mc\alpha_A\omega_\alpha^2$ ,  $k_{18} = 2 - \alpha_0^2$ ,  $k_{19} = \alpha_0\alpha_A$ ,  $k_{20} = \alpha_A^2$ , and  $k_{21} = \frac{L^3m}{3J_3}\alpha_A\omega_\alpha^2$ ; note that  $g_1(t)$ ,  $g_2(t)$  and  $g_3(t)$  are T-periodic, zero mean functions.

For the approximate model (5.18), the pull-back operation can be obtained directly using its original definition rather than through (5.13), since the *g* field does not involve the state vari-

ables. It can be easily shown that  $((\Phi_{0,\tau}^g)^* f) = f(\Phi_{0,\tau}^g)$ . With the definition of pull-back function  $(\Phi_{0,\tau}^g)^* f$ , we get

$$\dot{y}_{1} = \frac{m_{2}}{m_{1}}(y_{2}+h_{2})(y_{3}+h_{3}) - \frac{k_{1}}{m_{1}}(y_{1}+h_{1})^{2} + \frac{k_{2}}{m_{1}}(y_{2}+h_{2})^{2} + k_{4},$$
(5.19)

$$\dot{y}_{2} = -\frac{(k_{1}+k_{2})}{m_{2}}(y_{1}+h_{1})(y_{2}+h_{2}) +k_{5} - \frac{m_{1}}{m_{2}}(y_{1}+h_{1})(y_{3}+h_{3}),$$
(5.20)

$$\dot{y}_3 = -k_3(y_3 + h_3)^2 \operatorname{sgn}(y_3 + h_3) - k_6,$$
 (5.21)

where  $h_1(t) = \int_0^t g_1(\tau) d\tau$ ,  $h_2(t) = \int_0^t g_2(\tau) d\tau$ ,  $h_3(t) = \int_0^t g_3(\tau) d\tau$ , respectively. The term  $(y_3 + h_3)^2 \operatorname{sgn}(y_3 + h_3)$  in (5.21) will make the averaging overly complicated.

Define  $k_{22} = k_{17}k_{18} - \frac{3}{4}k_{17}k_{20} + k_{21}$ ,  $k_{23} = \frac{1}{2}k_{17}k_{19}$ ,  $k_{24} = \frac{1}{12}k_{17}k_{20}$ , and  $k_{25} = k_{17}k_{18} - \frac{3}{4}k_{17}k_{20} + k_{21} + \frac{1}{12}k_{17}k_{20}$ . We have

$$h_{3}(t) = \frac{\left\{-k_{22}\cos\omega_{\alpha}t + k_{23}\sin 2\omega_{\alpha}t - k_{24}\cos 3\omega_{\alpha}t + k_{25}\right\}}{\omega_{\alpha}} \\ \approx \frac{1}{\omega_{\alpha}} \left\{-k_{22}\cos\omega_{\alpha}t + k_{22}\right\},$$
(5.22)

since the size of  $k_{22}$  is typically two orders of magnitude bigger than those of  $k_{23}$ ,  $k_{24}$ ,  $|k_{22} - k_{25}|$ .

With such an approximation, the average dynamics (5.12) is given below,

$$\begin{aligned} \dot{z}_{1} &= z_{2}z_{3} + \bar{h}_{2}z_{3} + \bar{h}_{3}z_{2} + \bar{h}_{23} - k_{1}z_{1}^{2} \\ &- 2k_{1}\bar{h}_{1}z_{1} - k_{1}\bar{h}_{11} + k_{2}z_{2}^{2} + 2k_{2}\bar{h}_{2}z_{2}, \end{aligned}$$
(5.23)  

$$\dot{z}_{2} &= -z_{1}z_{3} - \bar{h}_{1}z_{3} + \bar{h}_{3}z_{1} + \bar{h}_{13} - (k_{1} + k_{2})z_{1}z_{2} + k_{5} \\ &- (k_{1} + k_{2})\bar{h}_{1}z_{2} - (k_{1} + k_{2})\bar{h}_{2}z_{1} - (k_{1} + k_{2})\bar{h}_{12}, \end{aligned}$$
(5.24)  

$$\dot{z}_{3} &= -k_{3} \bigg\{ \{\pi - 2\arccos(1 + \frac{\omega_{\alpha}}{k_{2}z_{3}})/\pi \} \\ &- 2k_{3}z_{3} \bigg\{ \frac{k_{22}}{\omega_{\alpha}} + \frac{2k_{22}}{\pi} \big[ \frac{1}{\omega_{\alpha}} \sqrt{1 - (1 + \frac{\omega_{\alpha}}{k_{2}z}z_{3})^{2}} \big] \\ &- \frac{1}{\omega_{\alpha}}\arccos(1 + \frac{\omega_{\alpha}}{k_{22}}z_{3}) \bigg\} \\ &- k_{3} \bigg\{ - 2\frac{k_{22}^{2}}{\omega_{\alpha}\pi} \big[ \frac{1}{4\omega_{\alpha}} \sin(2\arccos(1 + \frac{\omega_{\alpha}}{k_{22}}z_{3})) \big] \\ &- \frac{2}{\omega_{\alpha}} \sqrt{1 - (1 + \frac{\omega_{\alpha}}{k_{2}z}z_{3})^{2}} + \frac{3}{2\omega_{\alpha}}\arccos(1 + \frac{\omega_{\alpha}}{k_{22}}z_{3}) \big] \\ &+ \frac{3k_{22}^{2}}{2\omega_{\alpha}^{2}} \bigg\} \bigg\}$$
(5.25)

with  $\bar{h}_1 = \frac{1}{T} \int_0^T h_1 dt$ ,  $\bar{h}_2 = \frac{1}{T} \int_0^T h_2 dt$ ,  $\bar{h}_3 = \frac{1}{T} \int_0^T h_3 dt$ ,  $\bar{h}_{11} = \frac{1}{T} \int_0^T h_1^2 dt$ ,  $\bar{h}_{12} = \frac{1}{T} \int_0^T h_1 h_2 dt$ ,  $\bar{h}_{13} = \frac{1}{T} \int_0^T h_1 h_3 dt$ , and  $\bar{h}_{23} = \frac{1}{T} \int_0^T h_2 h_3 dt$ . Then the first-order average model is  $x(t) = \Phi_t^g(z(t))$ , according to Variation of constants formula [136].

With the tail-beat bias, frequency, and amplitude set to be  $20^{\circ}$ , 1 Hz, and  $15^{\circ}$ , respectively, we compare the turning radii and periods predicted by the model (5.18) and the corresponding average model. The parameters used in the simulation are based on those identified for the robotic fish as discussed later in the next section. The results are (0.26776 m, 59.7734 s) and (0.270276 m, 60.0873 s), respectively, which indicates that the first-order averaging is able to capture the original dynamics reasonably well. However, the resulting average model (5.23)-(5.25) is overly complex

to be useful for analysis or controller design, even though it is derived based on simplifications of the original dynamics.

#### 5.3.3 Classical Averaging

Considering its simplicity, we have also examined the possible use of the classical averaging method. The classical averaging can be applied to a slow system of the form

$$\dot{x} = \varepsilon f(t, x), \quad x(0) = x_0, \tag{5.26}$$

where  $\varepsilon$  is a small positive number and f(t,x) is T-periodic in t. The associated averaging system is

$$\dot{\mathbf{y}} = \varepsilon f_{av}(\mathbf{y}), \quad \mathbf{y}(0) = \mathbf{x}_0, \tag{5.27}$$

where

$$f_{av}(y) = \frac{1}{T} \int_0^T f(\tau, y) d\tau.$$
 (5.28)

**Theorem 1** [134] There exists a positive  $\varepsilon_0$ , such that for all  $0 < \varepsilon \le \varepsilon_0$ ,

1)  $x(t) - y(t) = O(\varepsilon)$  as  $\varepsilon \to 0$  on the time scale  $1/\varepsilon$  and

2) if the origin is asymptotically stable for  $f_{av}$ , then  $x(t) - y(t) = O(\varepsilon)$  as  $\varepsilon \to 0$  for all  $t \in \mathbb{R}_+$ and the system (5.26) possesses a unique periodic orbit which is locally asymptotically stable and belongs to an  $O(\varepsilon)$  neighborhood of the origin.

This averaging method is easy to implement; however, it cannot be directly applied to the robotic fish dynamics as the latter is not a slow system that satisfies Eq. (5.26). To illustrate this point, we have conducted simulation comparison of the original dynamic model and the averaged model with classical averaging, for the tail-beat patterns of  $\alpha_0 = 20^\circ$ ,  $\omega_{\alpha} = 2\pi$  rad/s, and  $\alpha_A = 15^\circ$ .

As shown in Fig. 5.1, the averaged model predicts a motion orbit that is significantly different from that predicted by the original dynamics.



Figure 5.1: The trajectories of robotic fish predicted by the original model and by the classical average model, respectively.

# 5.4 Computationally Eff cient Averaging with Scaled Forcing

#### 5.4.1 Proposed Averaging Approach

As demonstrated in the previous section, although the first-order averaging method for systems with oscillatory inputs can approximate well the original dynamics for robotic fish, the resulting model is overly complex for controller design and other applications. On the other hand, classical averaging is easy to implement with simple expressions, but it cannot be applied to the robotic fish model directly since the latter does not satisfy the slow dynamics requirement. Motivated by these
observations, we propose scaling the original forcing terms by some functions that are potentially dependent on the tail-beat parameters,  $\alpha_0$ ,  $\alpha_A$  and  $\omega_{\alpha}$ , and then conduct the classical averaging over the modified dynamics. Specifically, we modify the original system (5.5) to

$$\begin{cases}
\dot{x_1} = f_1(x) + K_{force} \cdot f_4(t), \\
\dot{x_2} = f_2(x) + K_{force} \cdot f_5(t), \\
\dot{x_3} = f_3(x) + K_{moment} \cdot f_6(t),
\end{cases}$$
(5.29)

where  $K_{force}$ ,  $K_{moment}$  are scaling functions dependent on tail-beat parameters. Note that the physical meanings of  $f_4(t)$  and  $f_5(t)$  are the projections of instantaneous hydrodynamic forces along the longitudinal and transverse directions in the body-fixed coordinates, so the scaling functions for them are the same. Using the second-order Taylor series expansion to approximate the sin  $\alpha$ and cos  $\alpha$  terms in (5.29), we obtain the averaged system as

$$\begin{cases}
\dot{x_1} = f_1(x) + K_{force} \left[ \frac{m}{12m_1} L^2 \omega_{\alpha}^2 \alpha_A^2 (3 - \frac{3}{2} \alpha_0^2 - \frac{3}{8} \alpha_A^2) \right], \\
\dot{x_2} = f_2(x) + K_{force} \left[ \frac{m}{4m_2} L^2 \omega_{\alpha}^2 \alpha_A^2 \alpha_0 \right], \\
\dot{x_3} = f_3(x) + K_{moment} \left[ \left( -\frac{m}{4J_3} L^2 c \omega_{\alpha}^2 \alpha_A^2 \alpha_0 \right) \right].
\end{cases}$$
(5.30)

We have conducted extensive simulations with different sets of  $\alpha_0$ ,  $\alpha_A$ , and  $\omega_\alpha$ , to identify the corresponding values of  $K_{force}$  and  $K_{moment}$  that would result in good match between the average model (5.30) and the original dynamics (5.5).

In the simulation, we use the parameters identified for a free-swimming robotic fish prototype as shown in Fig. 3.3, which has been reported in Chapter 3. The tail deflection angle  $\alpha$  is controlled by a servomotor through the tail shaft. The hydrodynamic coefficients for the body are  $C_D = 0.386$ ,  $C_L = 4.50$ , and  $K_D = 7.82 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ . For a tail-beat pattern (5.14) with  $\alpha_0 \neq 0$ , the trajectory of the robotic fish will converge to a circular orbit. For example, Figs. 5.2 and 5.3 show the simulated trajectory and heading of the robotic fish, respectively, based on the original dynamics, when the tail beats at  $\omega_A = 2\pi$  rad/s with bias  $\alpha_0 = 20^\circ$  and amplitude  $\alpha_A = 15^\circ$ . The trajectory is close to a circle with oscillatory ripples, as seen in the close-up in Fig. 5.2b. The average heading changes continuously with similar smallamplitude oscillations as seen in Fig. 5.3b. From the simulation, we can extract the turning radius and turning period (time taken to complete one turn) of the robot at the steady state.

With the average model (5.30), the trajectory of the robot converges to a circle free of ripples. Analysis of the stability properties for the average model will be presented in Section 5.5. For each set of  $(\alpha_0, \alpha_A, \omega_\alpha)$ , we have conducted blanket-search of parameters  $K_{force}$  and  $K_{moment}$ , such that the resulting average model would produce the best match in turning radius and period with the original dynamics.

In the simulation, we have used five tail-beat frequencies: 1, 2, 3, 4, 5 Hz; five bias angles: 10, 20, 30, 40, 50 degrees; and five amplitudes: 10, 15, 20, 25, 30 degrees, and find the optimal coefficients for all  $5 \times 5 \times 5 = 125$  combinations of these parameters. Fig. 5.4 shows the optimal coefficients for different combinations of tail-beat bias and amplitude when the tail beats at 1 Hz. The results for other frequencies are nearly identical to those in Fig. 5.4, and are thus not presented in the interest of brevity. From Fig. 5.4(a), we can see that the optimal force coefficient is nearly constant for different biases and amplitudes. On the other hand, as shown in Fig. 5.4(b), the optimal moment coefficient shows linear dependence on the bias angle and barely changes with the amplitude.

Fig. 5.5 shows how the scaling coefficients vary with the bias and the frequency for a given amplitude (15°); nearly identical results hold for the other amplitudes simulated. Again,  $K_{force}$  is nearly constant for different combinations of tail-beat bias and frequency, and  $K_{moment}$  shows



Figure 5.2: Simulated trajectory of the center of the robotic fish when the tail beats at 1 Hz with bias  $20^{\circ}$  and amplitude  $15^{\circ}$ : (a) The global path; (b) close-up view of the segment within the red rectangular region in (a).

linear dependence on the bias and independence of the frequency. Fig. 5.6 shows the obtained coefficients versus the frequency and the amplitude, for a fixed bias of 20°. In consistency with Figs. 5.4 and 5.5, we see that both coefficients are independent of the frequency and the amplitude. In addition, in all cases, the values of  $K_{force}$  are very close, as are the slopes of  $K_{moment}$  with



Figure 5.3: Simulated heading angle of the robotic fish when the tail beats at 1 Hz with bias  $20^{\circ}$  and amplitude  $15^{\circ}$ : (a) The heading angle trajectory; (b) close-up view of the segment within the red rectangular region in (a).

respect to the bias.

Based upon above observations, we get the force scaling coefficient by taking the average of



Figure 5.4:  $K_{force}$  and  $K_{moment}$  versus the tail-beat bias and amplitude. In all cases, the tail beats at 1 Hz.

all corresponding coefficients obtained in 125 individual simulations, which gives

$$K_{force} = 0.8485.$$

For each tail beating bias, we take the average of the optimal K<sub>moment</sub> for 25 different combinations



Figure 5.5:  $K_{force}$  and  $K_{moment}$  versus the tail-beat bias and frequency. In all cases, the tail beats with amplitude  $15^{\circ}$ .

of amplitude and frequency, as summarized in Table 5.1. Then the least-square-error fitting is used to identify the relationship between  $K_{moment}$  and the bias  $\alpha_0$ , resulting in

$$K_{moment} = -0.0074 + 0.4831 \alpha_0,$$



Figure 5.6:  $K_{force}$  and  $K_{moment}$  versus the tail-beat amplitude and frequency. In all cases, the tail beats with bias 20°.

where  $\alpha_0$  is expressed in radians.

Bias (°)	Average of moment coefficient
10	0.0796
20	0.1596
30	0.2424
40	0.33
50	0.416

Table 5.1: Average of moment coefficients for different bias angles.

#### 5.4.2 Numerical Evaluation of the Average Model

In order to evaluate the proposed averaging approach, we simulate the average model (5.30) with the obtained scaling functions. Fig. 5.7 shows the comparison of the turning radii and turning periods predicated by the original model and the average model, respectively, under different tailbeat biases, where the tail-beat frequency and amplitude are fixed at 1 Hz and 20°, respectively. We have deliberatively chosen cases of 25° and 35° biases at this stage (and the latter part in this section), which are not in the aforementioned 125 motion combinations, to provide independent validation for the proposed averaging approach. It can be seen that the average model is able to capture the original dynamics in the steady turning motion with high fidelity.

We then validate the effectiveness of the proposed averaging approach by comparing the cruising speeds predicted by the original and average models, when the tail beat bias  $\alpha_0 = 0$ . Note that  $\alpha_0 = 0$  has not been used in obtaining the scaling coefficients  $K_{force}$  and  $K_{moment}$ , and thus this new set of simulation provides a meaningful test for the averaging approach. For this simulation, we randomly pick some tail-beat frequencies and amplitudes, and the simulated velocities using the original and average models are shown in Table 5.2. It can be seen that the prediction error by the average model is less than 2% for all cases.

To further evaluate the generality of the proposed averaging method, we compare the original and average models for new sets of parameters. In particular, we double the weight of the robot (and the inertia of robot's body, correspondingly) from  $m_b = 0.311$  kg to  $m_b = 0.622$  kg,



Figure 5.7: Comparison of steady-state turning radii and periods predicted by the original and average models. In all cases, the tail-beat frequency is 1 Hz and amplitude is  $20^{\circ}$ .

Table 5.2: Surge velocities predicted by the original and average models, and the relative error by the average model.

Frequency (Hz)	Amplitude (°)	Original	Averaged	$E_{vel}$ (%)
3.1181	18.7749	0.112114	0.114017	1.70
1.0955	17.6312	0.036991	0.037648	1.78
1.8308	25.3103	0.088677	0.089746	1.21
1.1385	25.9040	0.056431	0.057084	1.16
1.2914	13.7375	0.033976	0.034661	2.0
3.4704	19.7953	0.131556	0.133695	1.62
3.0845	18.9117	0.111713	0.113599	1.69
1.9513	22.9263	0.085645	0.086837	1.39
3.8507	24.1873	0.178275	0.180581	1.29
1.1033	25.0937	0.052984	0.053632	1.22

while keeping the other parameters unchanged (note that the added mass/inertia are determined by robot's dimensions and independent of robot's mass/inertia). Then following the same procedure as described in the early part of this section, one could identify the best scaling functions for the

system, by matching the turning parameters (radius and period) from the original dynamic model and from the average model. We fix the tail-beat amplitude and frequency to be 20° and 1 Hz, respectively, and vary the tail-beat bias. We note that, instead of blanket-searching of parameters by running extensive simulations, the observations of behaviors for  $K_{force}$  and  $K_{moment}$  can be exploited to find these parameters in a significantly more efficient manner. In particular, we find the best scaling coefficients for the system when the bias is 10°, 20° and 30°, respectively, as shown in Table 5.3.

Bias (°)	Force coefficients	Moment coefficient
10	0.8780	0.2500
20	0.8980	0.4650
30	0.8980	0.610

Table 5.3: Best scaling coefficients for different bias angle. ( $m_b = 0.622$  kg in the simulation)

The force coefficient is then chosen to be  $K'_{force} = 0.891$  by averaging the coefficients. The moment coefficients can be obtained by a linear interpolation  $K'_{moment} = 0.0817 + 1.0313\alpha_0$ . Then the simulations are conducted with the updated force and moment coefficients and the results are shown in Figures 5.8 and 5.9. We can see that the predictions by average model match well those predicted by the original dynamic model.

To investigate the performance of the average model when the tail-beat parameters vary over time, in particular, under a feedback control context, we design a simple proportional controller for the original system. In the simulations, the proportional controller depends on the present error, which is defined as the difference between the orientation of the target relative to the robot center and the heading of the robot. The tail is programmed to beat at 1 Hz, with an amplitude of  $15^{\circ}$ , while the bias is updated by the proportional controller every five seconds. Fig. 5.10a shows the simulation trajectory of the robot, where it starts at the origin from rest and moves to the target location (2,2). The corresponding bias obtained from the feedback is shown in Fig. 5.10c. The



Figure 5.8: Simulation results from the original and average models (versus different tail-beat bias), when  $m_b = 0.622$  kg. In all cases, the tail-beat frequency is 1Hz and amplitude is 20°.

time trajectory of this bias is then applied to the averaged system, which predicts the robot trajectory as shown in Fig. 5.10b. With the same bias input, the terminal point with the original model is (1.9176,2.2430) while the robot trajectory with the average model stops at (2.0849,2.2803) after 200 s of simulation time. By comparing Fig. 5.10a and 5.10b, we can see that there is some modest error, which can be explained by that the average model does not account for the transients when the tail-beat parameters change. However, overall the average model is able to predict the trajectory of the robot.

#### 5.4.3 Experimental Evaluation of the Average Model

For model validation purpose, we have conducted experiments with the robot in forward swimming. We note that this forward swimming case ( $\alpha_0 = 0$ ) was not used in obtaining the scaling



Figure 5.9: Simulation results from the original and average models (versus different tail-beat amplitude), when  $m_b = 0.622$  kg. In all cases, the tail-beat frequency is 1Hz and bias is 20°.

functions, and thus this experimental result provides a significant validation for the averaging approach. Fig. 5.11 shows the experimental and simulation results by comparing the time that the robotic fish travels 50 cm long after it reaches steady speed, and we can see that the average model is able to not only capture the original dynamics but also predict the robot's motion with high fidelity.

Previous discussions have exclusively focused on the steady motion behavior of a robotic fish. It is of interest to investigate the performance of the average model in predicting the transient dynamics. For this purpose, we have conducted experiments to record the transient trajectories of the robot when it is started from at rest. A yellow LED is taped to the top of the robot as a marker and its motion has been video-recorded with the ambient light turned off. The robot's time trajectory, as shown in Fig. 5.12, is then extracted by applying background subtraction to the



Figure 5.10: Trajectory of the center of the robot when the tail-beat bias is updated by a proportional feedback controller law (a) The simulated trajectory using the original model; (b) The simulated trajectory using the average model; (c) The control input to both the original and average models.

images. Fig. 5.12(a) and (b) compare the model predictions of time-trajectories of the robot with the experimental measurements in X and Y coordinates, respectively. Fig. 5.12(c) provides the comparison of the trajectories in the XY-plane. It can be seen that the developed average model is able to capture not only the steady motion but also the transients.



Figure 5.11: Comparison between prediction from original dynamic model, average model and experimental measurement for forward swimming.

# 5.5 Analysis of the Average Model

It is of interest to analyze the average model (5.30) in terms of its stability properties. In particular, for a given tail-beat pattern, it is important to know whether the model admits an equilibrium, and if so, whether the equilibrium is unique and stable.

The equilibrium points can be obtained by setting  $\dot{\mathbf{x}} = 0$ . The highly nonlinear terms like  $\arctan(x_2/x_1)$  make it impossible to get an analytical solution. However, if we make the same assumptions as in Section 5.3.2,  $x_1 \gg x_2$  and  $x_1 > 0$ , then we can obtain unique, analytical solutions. In particular, with these assumptions, we can approximate  $\arctan(x_2/x_1) \approx (x_2/x_1)$  and



Figure 5.12: Comparison between predictions from the average model and experimental measurement for forward swimming (including transients): (a) time trajectory of *X*-coordinate of the robot; (b) time trajectory of the *Y*-coordinate of the robot; (c) path of the robot in the *XY*-plane. For both the experiments and the simulation, the amplitude and frequency of the tail beat are fixed at  $15^{\circ}$  and 0.9 Hz, respectively.

 $(x_1^2 + x_2^2) \approx x_1^2$ . The equilibrium point is then solved as

$$x_{1} = \sqrt{\frac{K_{force}mL^{2}\omega_{\alpha}^{2}\alpha_{A}^{2}(3-\frac{3}{2}\alpha_{0}^{2}-\frac{3}{8}\alpha_{A}^{2})}{6\rho SC_{D}}}$$

$$x_{2} = \frac{K_{force}}{2\rho S(C_{D}+C_{L})}\sqrt{\frac{6m\rho SC_{D}L^{2}\omega_{\alpha}^{2}\alpha_{A}^{2}\alpha_{0}^{2}}{K_{force}(3-\frac{3}{2}\alpha_{0}^{2}-\frac{3}{8}\alpha_{A}^{2})}}$$

$$+\frac{2m_{1}}{\rho S(C_{D}+C_{L})}\sqrt{\frac{K_{moment}mL^{2}c\omega_{\alpha}^{2}\alpha_{A}^{2}\alpha_{0}}{4K_{D}}}$$

$$x_{3} = -\sqrt{\frac{K_{moment}mL^{2}c\omega_{\alpha}^{2}\alpha_{A}^{2}\alpha_{0}}{4K_{D}}}$$

Next we analyze the stability property of the equilibrium for the model (5.30). Due to the highly nonlinear nature of the model, analysis of global asymptotic stability is difficult. Instead, we focus on the local stability in the neighborhood of the equilibrium. We rewrite Eq. (5.30) in a compact form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}),$$

where  $\mathbf{x} = [x_1, x_2, x_3]^T$  and  $\mathbf{u} = [\alpha_0, \alpha_A, \omega_\alpha]^T$ . We then linearize the system

$$A = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$
(5.31)

to get the Jacobian matrix A, the elements of which are

$$\begin{aligned} a_{11} &= -\frac{c_1}{m_1} \frac{x_1^2}{\sqrt{x_1^2 + x_2^2}} + \frac{c_2}{m_1} \frac{x_1 x_2}{\sqrt{x_1^2 + x_2^2}} \arctan(\frac{x_2}{x_1}) \\ &\quad -\frac{c_1}{m_1} \sqrt{x_1^2 + x_2^2} - \frac{c_2}{m_1} \sqrt{x_1^2 + x_2^2} \frac{x_2^2}{x_1^2 + x_2^2}, \\ a_{12} &= \frac{m_2}{m_1} x_3 - \frac{c_1}{m_1} \frac{x_1 x_2}{\sqrt{x_1^2 + x_2^2}} + \frac{c_2}{m_1} \sqrt{x_1^2 + x_2^2} \arctan(\frac{x_2}{x_1}) \\ &\quad + \frac{c_2}{m_1} \frac{x_2^2}{\sqrt{x_1^2 + x_2^2}} \arctan(\frac{x_2}{x_1}) + \frac{c_2}{m_1} \sqrt{x_1^2 + x_2^2} \frac{x_1 x_2}{x_1^2 + x_2^2}, \\ a_{13} &= \frac{m_2}{m_1} x_2, \\ a_{21} &= -\frac{m_1}{m_2} x_3 - \frac{c_1}{m_2} \frac{x_1 x_2}{\sqrt{x_1^2 + x_2^2}} - \frac{c_2}{m_2} \sqrt{x_1^2 + x_2^2} \arctan(\frac{x_2}{x_1}) \\ &\quad - \frac{c_2}{m_2} \frac{x_1^2}{\sqrt{x_1^2 + x_2^2}} \arctan(\frac{x_2}{x_1}) + \frac{c_2}{m_2} \sqrt{x_1^2 + x_2^2} \frac{x_1 x_2}{x_1^2 + x_2^2}, \\ a_{22} &= -\frac{c_1}{m_2} \frac{x_2^2}{\sqrt{x_1^2 + x_2^2}} - \frac{c_2}{m_2} \sqrt{x_1^2 + x_2^2} \arctan(\frac{x_2}{x_1}) \\ &\quad - \frac{c_1}{m_2} \sqrt{x_1^2 + x_2^2} - \frac{c_2}{m_2} \sqrt{x_1^2 + x_2^2} \frac{x_1^2}{x_1^2 + x_2^2}, \\ a_{23} &= -\frac{m_1}{m_2} x_1, \\ a_{31} &= 0, \\ a_{32} &= 0, \\ a_{33} &= -2c_4 x_3 \text{sgn}(x_3). \end{aligned}$$

By examining the Hurwitz property of *A*, we can know whether the averaged system is asymptotically stable at its equilibrium points. Note that  $a_{33}$  is negative except when  $x_3 = 0$ . Since both  $a_{31}$ 

and  $a_{32}$  are 0, we only need to check the Hurwitz property of a two-two matrix

$$A' = \left[ \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right],$$

the characteristic equation of which is

$$\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0.$$

In order to show that A' is a Hurwitz matrix, we need to have  $a_{11} + a_{22} < 0$  and  $a_{11}a_{22} - a_{12}a_{21} > 0$ . It is easy to see that  $a_{11} + a_{22} = -(3c_1 + c_2)\sqrt{x_1^2 + x_2^2}$ , and thus is less than 0 except when  $x_1 = x_2 = 0$ . By canceling the terms  $\frac{m_2}{m_1}x_3$  and  $-\frac{m_1}{m_2}x_3$  in view of the negligible coupling terms in (8)-(9), we have  $a_{11}a_{22} - a_{12}a_{21} = 1/(m_1m_2) \left[ (2c_1^2 + 2c_1c_2)(x_1^2 + x_2^2) + 2c_2^2(x_1^2 + x_2^2) \arctan^2(\frac{x_2}{x_1}) \right]$ , which is always positive.

From the above analysis, we show that the linearization matrix *A* is Hurwitz at an equilibrium point independent of its specific values, which proves the local asymptotic stability of the average model given a particular set of control input ( $\alpha_0, \alpha_A, \omega_\alpha$ ).

## 5.6 Application to Semi-analytical Modeling of Steady Turning

#### 5.6.1 Semi-analytical Modeling Framework

The proposed averaging approach is instrumental in several ways. For example, it facilitates model-based controller design, as was demonstrated in [139], where a backstepping approach is developed for tracking control of robotic fish. In this section, we present another application of the average model, where it is used to gain analytical insight into steady turning of robotic fish.

Section 5.2 provides a complete dynamic model for a tail-actuated robotic fish, and one can extract turning information by observation from simulated trajectories as in [14]. However, these simulations are time-consuming and cannot provide analytical insights into how the turning parameters are related to system parameters or tail-beat parameters. In our group's prior work [106], a modeling framework was proposed for computation of steady-state turning motion given asymmetric, periodic body/tail deformation of a robotic fish. In that approach, it is postulated that the two key parameters of turning motion, the radius and the period, can be obtained by solving the implicit force and moment balance equations for the averaged, steady-state motion. However, the modeling framework there adopts two major assumptions: 1) the body direction is aligned with the motion direction, i.e. the angle of attack remains to be zero during the robot's motion; 2) the classical averaging method can be directly applied to robotic fish dynamics (which, as we show in Section III-C, is generally not true). In this section, we apply the average model obtained with the proposed approach in this chapter, to develop an semi-analytical modeling framework understand-ing the turning behavior of robotic fish.

Fig. 5.13 illustrates a robotic fish undergoing a steady turning motion.  $[\mathbf{X}, \mathbf{Y}, \mathbf{Z}]$  and  $[\mathbf{x}, \mathbf{y}, \mathbf{z}]$  denote the aforementioned inertial coordinates and body-fixed coordinates, respectively.  $[\mathbf{x}', \mathbf{y}', \mathbf{z}']$  denote the moving coordinates attached to the fish body, where  $\mathbf{x}'$  points to the tangential direction of the circular path.  $\alpha$  is the aforementioned oscillating input (tail angle).

With the averaging method developed in Section IV, the time-explicit oscillating terms are replaced with constant inputs, particularly, the tail-beat parameters,  $\alpha_0$ ,  $\alpha_A$  and  $\omega_{\alpha}$  in (5.30). Then the robotic fish can be treated as a rigid body under force and moment balances. The averaged



Figure 5.13: Top view of the tail-actuated robotic fish undergoing "steady" turning motion.

forces along and perpendicular to the body direction are

$$\begin{cases} F_{\mathbf{x}} = K_{force} \left[ \frac{m}{12m_1} L^2 \omega_{\alpha}^2 \alpha_A^2 (3 - \frac{3}{2} \alpha_0^2 - \frac{3}{8} \alpha_A^2) \right], \\ F_{\mathbf{y}} = K_{force} \left[ \frac{m}{4m_2} L^2 \omega_{\alpha}^2 \alpha_A^2 \alpha_0 \right]. \end{cases}$$
(5.32)

Then the forces along the radial and tangential directions (with respect to the steady circular path) are

$$\begin{cases} F_{\mathbf{x}'} = F_{\mathbf{x}} \cos \beta + F_{y} \sin \beta, \\ F_{\mathbf{y}'} = -F_{\mathbf{x}} \sin \beta + F_{y} \cos \beta. \end{cases}$$
(5.33)

As the robot is under steady turning, we have the following force and moment balance equa-

tions:

$$\begin{cases}
F_{\mathbf{x}'} - F_D = 0, \\
F_{\mathbf{y}'} - F_L + m_b \omega^2 R = 0, \\
M_{\mathbf{z}} - M_D = 0,
\end{cases}$$
(5.34)

where  $M_z$  is the moment relative to the center of mass caused by the tail motion, which is equal to  $K_{moment} \left[ \left( -\frac{m}{4J_{bz}} L^2 c \omega_{\alpha}^2 \alpha_A^2 \alpha_0 \right) \right]$ . The unknowns in (5.34) are angular speed  $\omega$  (which is related to turning period  $T_p$  by  $T_p = \frac{2\pi}{\omega}$ ), turning radius R, and the angle of attack  $\beta$ . Therefore, these turning parameters can be directly obtained by solving the algebraic equation (5.34), instead of simulating the differential equation (5.30) and observing its steady-state orbit, as done in Section IV-B and IV-C. Note that Eq. (5.34) can only be numerically solved (hence explaining our term "semi-analytical"), due to its transcendental nature. In particular, we limit the values of  $\beta \in [0^\circ, 20^\circ]$  and use the *fsolve* command in Matlab to get the solution.

#### 5.6.2 Experimental Evaluation

To validate the steady turning framework, we compare the model predictions and experimental results. The experiments were conducted in our previous work [15]. As shown in Figures 5.14 - 5.16, the steady turning framework can capture the original dynamic model very well and predict the robot's turning behavior in all cases. Here the turning radius and turning period under "the average model" are obtained by solving the algebraic equations discussed in Section VI-A, instead of through integration of the differential equations as in Section IV. We note that the tail-beat motion here is numerically different (for example, tail-beat frequency is 0.9 Hz) from those in obtaining the scaling function in Section 5.4, and thus these results provide strong support for the proposed approach.



Figure 5.14: Comparison between model predictions and experimental measurement of turning behavior versus tail-beat bias.



Figure 5.15: Comparison between model predictions and experimental measurement of turning behavior versus tail-beat frequency.

# 5.7 Nonlinear Controller Design

One promising application of robotic fish is their use as mobile sensing platforms in aquatic environment to monitor water quality and track oil spills. To execute such tasks, robotic fish will need



Figure 5.16: Comparison between model predictions and experimental measurement of turning behavior versus tail-beat amplitude.

be able to track a given trajectory or a set of target points. A fish-like swimming robot is a system that is difficult to control due to its nonlinear, nonholonomic nature.

Most prior work on the control of carangiform robotic fish proposed to use model-free controllers, such as fuzzy logic control [140, 141]. These approaches are useful for engineering application, but they are not amenable to stability analysis, which is fundamental to understanding the closed-loop system's behavior. In this work, we investigate target tracking control problem for a tail-actuated robotic fish, based on the averaged robot dynamic model, as reported in Section 5.4 in this Chapter. To simplify the problem and emphasize the controller design, the tail-beat frequency and amplitude are fixed and the only control input is the tail-beat bias  $\alpha_0$ .

#### 5.7.1 System Description

Fig. 5.17 shows the target tracking control schematic diagram. Here,  $(x_s, y_s)$  and (x, y) denote the locations of the target and the robot's center of mass, respectively, relative to the inertial coordinate

frame.



Figure 5.17: Target tracking control schematic diagram.

Define *r* as the distance between  $(x_s, y_s)$  and (x, y), and  $\theta$  as the orientation error, specified by the angle between the robotic fish heading direction and the line connecting the body center to the target. The pair  $(r, \theta)$  can be considered as the target location in the body-fixed polar coordinates. We take  $\theta$  and *r* as auxiliary state variables, which are expressed as

$$\begin{cases} r = \sqrt{x_e^2 + y_e^2}, \\ \theta = \psi - \varphi, \end{cases}$$
(5.35)

where  $y_e = y_s - y_s$ ,  $x_e = x_s - x_s \phi = \arctan(y_e/x_e)$ . The target tracking is achieved if *r* converges to zero, which is the goal of the controller's design in this chapter. The dynamic equations for the

two new variables  $\theta$  and *r* can be written as

$$\begin{cases} \dot{r} = -V_{Cx}\cos\theta + V_{Cy}\sin\theta, \\ \dot{\theta} = \frac{V_{Cx}}{r}\sin\theta + \frac{V_{Cy}}{r}\cos\theta + \omega_z. \end{cases}$$
(5.36)

#### 5.7.2 Hybrid Controller Design

In order to drive r to zero (or the neighborhood of zero), consider a Lyapunov function

$$V_1 = \frac{1}{2}r^2.$$
 (5.37)

However, we cannot show  $\dot{V}_1$  negative no matter what kind of control input is provided. Define a second Lyapunov equation as

$$V_2 = V_1 + \frac{1}{2}z_1^2, \tag{5.38}$$

where  $z_1 = k_r r - V_{Cx} \cos \theta + V_{Cy} \sin \theta$ , and  $k_r$  is a positive coefficient. We check the value of  $\dot{V}_2$ and it turns out  $\dot{V}_2$  would be negative if there exists an  $\alpha_0$  satisfying  $f(\alpha_0) = 0$ , where

$$f(\alpha_0) = A\alpha_0^2 + B\alpha_0 + C = 0 \tag{5.39}$$

and,

1

$$\begin{split} A &= \frac{1}{2} \bar{c}_1 k_f \omega_\alpha^2 \alpha_A^2 \cos \theta, \\ B &= \bar{c}_1 k_f \omega_\alpha^2 \alpha_A^2 \sin \theta, \\ C &= \bar{c}_1 k_f \omega_\alpha^2 \alpha_A^2 \cos \theta (1 - \frac{1}{8} \alpha_A^2) \\ &+ (k_1 + k_r) (-V_{Cx} \cos \theta + V_{Cy} \sin \theta) + (k_1 k_r) r \\ &- \cos \theta [V_{Cy} \omega + f_u (V_{Cx}, V_{Cy})] + \sin \theta [-V_{Cx} \omega + f_v (V_{Cx}, V_{Cy})] \\ &+ (V_{Cx} \sin \theta + V_{Cy} \cos \theta) (\frac{V_{Cx}}{r} \sin \theta + \frac{V_{Cy}}{r} + \omega). \end{split}$$

We can show that there exists a solution of Eq. (5.39) when orientation error  $|\theta| < 45^{\circ}$ , which brings in the second controller design.

As shown in [15], the robotic fish converges to a circle when the bias is nonzero, and the larger the bias, the smaller the radius of the circle will be. If the orientation angle error is not in the interval where the approaching controller works, we set the bias to the allowable maximum, which indicates the most effective turning. The sign of the bias will be determined by the sign of  $\theta$ . The turning controller can be described as

$$\alpha_0 = \alpha_{0max} \operatorname{sgn}(\theta), \tag{5.40}$$

where  $\alpha_{0max}$  is a positive constant which denotes the allowable maximal tail-beat bias.

To avoid, possible frequent switching between two controllers, a hysteretic switching strategy is adopted by introducing two thresholds  $\theta_1$  and  $\theta_2$ , as shown in Fig. 5.18.



Figure 5.18: Switching strategy between two controllers.

#### 5.7.3 Simulation and Experimental Results

Extensive simulations have been conducted to verify the effectiveness of the proposed method. Fig. 5.19 shows the results of target tracking from different initial locations but the same initial orientations.



Figure 5.19: Simulation results with the target located at the origin and the robotic fish starting from different initial conditions (denoted by solid blue squares). The locations of controller switching are denotes by green diamonds.

To further verify the proposed method, experiments have been conducted and the setup is shown in Fig. 5.20. Two LED makers are fixed on the fish body and a camera is installed on the ceiling above a large indoor water tank. A computer which is connected to the camera is used to process the acquired images of the two LED markers, to obtain the state estimates for the robot. The robotic fish can communicate with the computer via Zigbee. The proposed control scheme is implemented with Visual C++ development tools. The control input is calculated based on the state information obtained by camera every oscillating period and is then sent to robotic fish via the Zigbee wireless module. We select several target points randomly in the field of view of camera and the target point is updated when r reaches 0.02 m. Fig. 5.21 shows final experimental results which validate the effectiveness of the proposed hybrid controller.

Results from this section have been published in Proceeds of 2013 ASME Dynamics Systems and Control Conference [139]. Beyond my advisor, the other author of this work Songlin Chen is an associated professor at Harbin Institute of Technology, Harbin, China, and visited our lab at 2012. Dr. Chen derived most of the back-stepping based hybrid controller and I was in charge of experimental validation and part of the theoretical derivations.

## 5.8 Conclusions

In this chapter, a novel averaging method has been presented for dynamics of a carangiform robotic fish model actuated through a tail. The merit of the approach lies in its simplicity, with the resulting average model amenable to analysis and control design. In particular, the proposed method builds upon classical averaging, but with the forcing terms in the original dynamic model scaled appropriately. It is found that the scaling functions take simple forms – constant for the force scaling while linear in bias for the moment scaling. The validity of the approach has been evaluated



Figure 5.20: Experimental setup.

with simulation and experimental results, where steady-state and transient behaviors predicted by the average model and the original model, respectively, are compared under various scenarios not used in obtaining the scaling functions. We have further conducted analysis of the average model in terms of the existence and stability of its equilibria. As an application of the proposed average model, we have demonstrated a new framework for studying steady turning behavior of a robotic fish without having to simulate the differential equations for the original dynamic model or the



Figure 5.21: Experimental results on tracking multiple target points continuously.

average dynamic model.

In the future work, we will explore analytical insight as to why the scaling functions take the specific simple forms. We will also seek to extend the approach to the dynamic model of robotic fish actuated with a flexible tail [132], as well as explore the validity of the approach for a wider class of systems with oscillatory inputs.

# Chapter 6

# Environmental Monitoring with Robotic Sensor Network: Sampling and Reconstruction of Low-rank Matrix Fields

# 6.1 Introduction

Monitoring aquatic environments is of growing interest to public health, marine biology and ecosystem sustainability. In recent years, aquatic environments have been faced with a variety of threats from climate change and industrial pollutions [142–144]. For instance, there was an oil spill in Kalamazoo River, Michigan in 2010, when a pipeline burst and oil flowed into the river [145]. In another incident, more than 300,000 gallons of diesel fuel spilled in Arthur Kill, New Jersey after a storage tank was ruptured [146]. During and after these accidents, monitoring the water quality and detecting pollutants in the water is a vital task [147, 148].

Manual sampling using hand-held devices, is still the most common practice in monitoring aquatic environment. This approach is labor-intensive and has difficulty in capturing large-scale spatially distributed phenomena of interest. An alternative approach is in-situ sensing on fixed buoys [56]. However, since buoyed sensors cannot move around, they have limited adaptability in monitoring and forecasting dynamic aquatic processes. There is a growing interest in using underwater sensor platforms like autonomous underwater vehicles (AUVs) [74, 149, 150] and sea gliders [151] to monitor the environment. However, it is not applicable to deploy many of them due to their high manufacturing and operational cost. In this chapter, we propose to use inexpensive, highly adaptive and maneuverable robotic fish platforms to sample and reconstruct spatiotemporal aquatic fields.

Sampling and reconstruction of physical fields using mobile sensor networks have received significant interest. Early work has dealt with optimal placement of static sensors to reduce the uncertainty [152], or guidance of one mobile sensor to reduce the field reconstruction error, given the measurements of the a static sensor network [153]. Field reconstruction using mobile sensor networks has been studied recently by several groups [154–156].

In this work, we investigate data-adaptive path planning schemes for wireless networks of robotic sensor platforms. Traditional sampling theory deals with data collection processes that are independent of the target map to be estimated, aside from some possible prior knowledge and/or assumptions. On the other hand, sequential, adaptive data collection strategies [157, 158] have gained increasingly interests due to the fact that the existing observations can be used to guide the future actions to achieve more efficient solutions.

Field reconstruction using robotic sensor networks has been extensively studied, for fields modeled by Gaussian processes [154, 157–159]. In [157], a data-driven adaptive sampling algorithm is presented. The next sample locations are selected such that the closest neighbors are able to communicate with each other while the distances between the robots are maximized at the same time. It also has other rules such as taking samples in area where present sampling density is lower and avoiding areas where measurements. In [158], the authors present an adaptive sampling algorithm for multiple autonomous underwater vehicles by assuming that field variables vary linearly with the location. A cost function is proposed to measure the dissimilarity between current sampling and the next sampling, and the algorithm selects the new sampling locations to minimize the cost function. In [21], researchers present an adaptive sampling framework to scan and reconstruct a Gaussian temporal-spatial field using robotic fish sensor networks. In particular, these robots are controlled to work collaboratively in the form of a swarm to sense the environment in a series of carefully chose rendezvous regions. While all these approaches have their own merits, in real-world applications, the dynamics of the processes of interest could be very complicated and sometimes kernel modeling-based techniques may not provide adequate performance.

On the other hand, Compressive Sensing (CS) [160, 161] techniques have been applied to field monitoring, as they allow the reconstruction of a field with a subset of samples and hence save time and energy. In [162], trajectory planning algorithms based on the standard CS paradigm are presented for robots with a variety of mobility patterns, including Random Walk, Random Traveling Salesman, and Randomized Boustrophedon. The target is to minimize the cost function (i.e., traveling distance) by comparing these motion patterns and recovering the data using CS. Similar work has been reported in [163, 164], where the authors assume that the data has a sparse structure and come up with an optimal sampling solution by leveraging CS techniques. The major drawback of these techniques is that they do not take into account the underlying data structure obtained or estimated using real-time measurements, which makes them not amendable to implementing adaptive sampling strategies.

This work presents a novel adaptive sampling framework to guide mobile robots to conduct samplings more efficiently in an environmental field of interest, while maintaining the accuracy of the recovered data. We consider a two-dimensional space (for example, the horizontal plane at a certain depth), where the field can be represented by a low-rank matrix. The idea is to successively span the sampling area and reconstruct the field analytically. The methods are illustrated in simulations based on the temperature data collected in a large indoor water tank. The results show that the proposed adaptive sampling scheme is more computationally efficient and requires shorter travel distances than an existing low-rank matrix completion scheme [165].

The remainder of the chapter is organized as follows. First, the problem formulation is described in Section 6.2. An matrix completion approach using Augmented Lagrange Method (MC-ALM) is reviewed in Section 6.3. Our proposed adaptive sampling and reconstruction framework is discussed in Section 6.4. In Section 6.5, simulation results using real trace-driven data are presented to compare the performances of our proposed approach and the MC-ALM method. Discussions of weaknesses of the proposed method and potential solutions are presented in Section 6.6. Finally, concluding remarks are provided in Section 6.7.

#### 6.2 **Problem Formulation**

The goal of this work is to develop an active mobile sensing approaching using a group of mobile robots (for example, robotic fish), to recover an unknown field with minimal energy consumption. Fig. 6.1a shows the field of oil distribution on the the surface of Kalamazoo River following the 2010 spill. Traditional sampling methods suggest uniform sampling according to the Shannon sampling theorem to recover the (discretized) field in Fig. 6.1b. While the latter approach is able to provide the data field with high resolution, it could waste a significant amount of sensing resources, because many measurements collected might not provide any additional information. CS for sparse matrices and Matrix Completion (MC) [166] for low-rank matrices are two approaches for recovering the field of interest by collecting only a few *random* measurements, as illustrated in Fig. 6.1c. MC is able to reconstruct the data with good fidelity, however, random samples do not accommodate the mobility constraints of the robots and makes them unsuitable to implement adaptive sampling strategies.



(a)



(b)



Figure 6.1: Oil spill in the Kalamazoo River. 2010 (Photo Credit: National Wildlife Federation) (a) Oil map on the surface of Kalamazoo River; (b) discrete uniform sampling; (c) random samplings

In this work, we assume that the original field can be represented with a matrix as illustrated in Fig. 6.1b, where the field values within each grid are identical. Matrix completion is a very useful tool in many applications, e.g., machine learning [167], control [168], and computer vision [169]. It is often reasonable to assume that the matrix to recover is of low rank. The problem is formulated as: given a number of mobile robotic sensing platforms (i.e. robotic fish [1,15]), how to drive those robots to sample the low-rank matrix and recover the missing entries with high accuracy and low energy consumptions.

# 6.3 Matrix Completion Using the Augmented Lagrange Multiplier Method

In this section, for the purpose of comparison, we first provide a summary of an existing algorithm that solves the matrix completion problem using Augmented Lagrange Multiplier (MC-ALM) [165].

The general matrix completion problem can be expressed as

$$\min_{A} \|A\|_{*}, \quad \text{subject to} \quad A_{ij} = D_{ij}, \quad \forall (i,j) \in \Omega$$
(6.1)

where  $\|\cdot\|_*$  denotes the nuclear norm of a matrix (sum of its singular values),  $\Omega$  is the set of indices of samples, and *D* denotes the original low-rank matrix. It has been shown that most matrices *A* of low rank *r* can be perfectly recovered by solving the following optimization problem [166], provided the number *p* of samples satisfies  $p \ge Crn^{6/5} \ln n$  for some constant *C*. This bound has then been improved by the work of others [170, 171]. In [165], Lin *et al.* proposed to use MC-ALM algorithm to solve a robust principle component analysis problem and extend it to complete
low-rank matrices. The problem is re-formulated as

$$\min_{A} \|A\|_*, \quad \text{subject to} \quad A+E=D, \quad \pi_{\Omega}(E)=0, \tag{6.2}$$

where  $\pi_{\Omega} : \mathbb{R}^{m \times n} \to \mathbb{R}^{m \times n}$  is a linear operator that keeps the entries in  $\Omega$  unchanged and sets those outside  $\Omega$  zeros. Since the error matrix *E* will compensate for the unknown entries of *D*, the unknown entries of *D* are simply set as zeros. Then the augmented Lagrangian function of (6.2) is given by [165]:

$$L(A, E, Y, \mu) = \|A\|_{*} + \langle Y, D - A - E \rangle + \frac{\mu}{2} \|D - A - E\|_{F}^{2},$$
(6.3)

where  $\|\cdot\|_F$  is the Frobenius norm,  $\mu$  is a positive scalar, and *Y* denotes the optimal Lagrange multiplier. Following [165], the MC-ALM approach is described in Algorithm 1, where *E* is updated with the constraint  $\pi_{\Omega}(E) = 0$  enforced when minimizing  $L(A, E, Y, \mu)$ .

### Algorithm 1: Matrix Completion via ALM Method [165]

**Inputs**: Observation samples  $D_{i,j}$ ,  $(i, j) \in \Omega$ , of matrix  $D \in \mathbb{R}^{m \times n}$ .

1 
$$Y_0 = 0; E_0 = 0; \mu_0 > 0; \rho > 1; k = 0;$$

- 2 while not converged do
- 3 // Lines 4-5 solve  $A_{k+1} = \arg \min_A L(A, E_k, Y_k, \mu_k)$ .

4 
$$(U,S,V) = svd(D - E_k + \mu_k^{-1}Y_k);$$

- 5  $A_{k+1} = UT_{\mu_k^{-1}}[S]V^T.$
- 6 // Line 7 solves  $E_{k+1} = \arg \min_{\pi_{\Omega}(E)=0} L(A_{k+1}, E, Y_k, \mu_k).$
- 7  $E_{k+1} = \pi_{\Omega}(D A_{k+1} + \mu_k^{-1}Y_k).$
- 8  $Y_{k+1} = Y_k + \mu_k (D A_{k+1} E_{k+1}).$
- 9 Update  $\mu_k$  to  $\mu_{k+1}$ .
- 10  $k \leftarrow k+1$ .
- 11 end while

**Output**:  $(A_k, E_k)$ .

 $T_{\mathcal{E}}[x]$  is the soft-thresholding operator:

$$T_{\varepsilon}[x] = \begin{cases} x - \varepsilon, & \text{if } x > \varepsilon, \\ x + \varepsilon, & \text{if } x < -\varepsilon, \\ 0, & otherwise. \end{cases}$$

# 6.4 Analytical Reconstruction via Successive Expansion of Sampling Area

A fundamental requirement for the low-rank MC algorithm described in the previous section is that the original matrix is sampled randomly. Such a requirement is difficult to satisfy for mobile robots due to their mobility constraints. In this section, we propose a new strategy for sampling and field-reconstruction by successively growing sub-matrices of the original matrix.

The algorithm is illustrated in Fig. 6.2. For each iterations, new samples (indicated by blue crosses in Fig. 6.2a) are taken to recover the new row and new column. For instance, consider the case where a matrix  $M \in \mathbb{R}^{(k+1)\times(k+1)}$  needs to be recovered fully based on its known sub-matrix  $M' \in \mathbb{R}^{k\times k}$  and some new samples in its outer row and column. For ease of explanation, we assume that the rank of M is equal to the rank of M', which is denoted by r, where  $r \leq k$ . M' thus has r linearly independent columns that can be used as a basis B for representing any other column  $\vec{v}$ . In particular, we can represent  $\vec{v}$  through the set of coefficients with respect to the basis B:

$$\operatorname{Rep}_{B}(\vec{v}) = \begin{pmatrix} c_{1} \\ c_{2} \\ \vdots \\ c_{r} \end{pmatrix}$$

where  $B = \langle \vec{\beta}_1, ..., \vec{\beta}_r \rangle$  and  $\vec{v} = c_1 \vec{\beta}_1 + c_2 \vec{\beta}_2 + \cdots + c_r \vec{\beta}_r$ . In *M*, those basis column vectors are also independent from each other and the rank of *M* is *r*, thus the columns corresponding to *B* will form a basis for  $M \in \mathbb{R}^{(k+1)\times(k+1)}$  and can be used to recover other columns with  $\operatorname{Rep}_B(\vec{v})$ . The same discussion applies to the rows. The sampling and reconstruction algorithm is summarized in

#### Algorithm 2.

### Algorithm 2: Matrix Completion via Analytical Reconstruction

**Inputs**: Rank *r* of the original matrix *r* 

- 1 Sample the first  $r \times r$  matrix by initializing k = r
- 2 while the dimension of the recovered matrix is less than that of the original field, do
- 3 // Lines 4-12 reconstruct M
- 4 Find r linearly independent columns as the basis  $B_{col}$
- 5 Take new samples (on the outer row) in those r columns to form the new basis  $B'_{col}$
- 6 Express the other columns *i* as a linear combination of  $B_{col}$  with  $(c_{i1}, \ldots c_{ir})^T$
- 7 Recover the outer row by multiplying coordinates with new basis  $B'_{col}$
- 8 Find *r* linearly independent rows as the basis  $B_{row}$
- 9 Take new samples (on the outer column) in those r rows to form the new basis  $B'_{row}$
- 10 Express the other rows *j* (including the outer row) as a linear combination of *B*<sub>row</sub> with  $(c_{i1}, \dots c_{ir})^T$
- 11 Recover the outer column by multiplying coordinates with new basis  $B'_{row}$
- 12 Update k to (k+1)
- 13 end while

Output: Recovered matrix.

## 6.5 Trace-Driven Simulations

We evaluate the performance of the MC-ALM method and our proposed algorithms through tracedriven simulations. <sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Trace driven simulation is getting the actual data from a real experiment and feeding it to the simulation



Figure 6.2: Sequential reconstruction of field of interest. (a) Successive expansion of sampling area, where the new samples in each iteration are only taken from the outer row and column; (b) Illustration of the sampling strategy with the oil spill example.

### 6.5.1 Measurement of temperature f eld in a water tank

Experiments are first conducted to measure the distribution of water surface temperature in a water tank, as illustrated in Fig. 6.3. A 1000W water heater (the red dot point in Fig. 6.3) is fixed in the tank. The heater is powered on five hours before the measurements and remains on through the whole experiment, so that the distribution of temperature reaches its equilibrium and is considered to be time-invariant during the sampling.



Figure 6.3: Sketch of the experimental setup for measuring the water-temperature distribution near the surface.

The 2-D scalar water-temperature field is then described by a matrix  $A \in \mathbb{R}^{24 \times 24}$  with singular value decomposition:

$$A = USV^T$$
,  $S = \operatorname{diag}(\sigma_1, \sigma_2, \dots, \sigma_{24})$ ,

where the singular values are ordered in a decreasing order. To test the matrix completion algorithms, we approximate A with a low rank matrix  $\tilde{A}$  by setting

$$\tilde{A} = U\tilde{S}V^T$$
,  $\tilde{S} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_6, 0, \dots, 0)$ ,

Fig. 6.4 shows the sensing field  $\tilde{A}$  with pixels being scaled between 0 and 255.



Figure 6.4: Scalar field of water-temperature.

### 6.5.2 Simulation results

In this subsection we evaluate the performance of MC-ALM and our proposed algorithms using the collected temperature data.

For the MC-ALM method, 50% measurements are randomly taken to recover the field. Fig. 6.5 shows the random sampling locations and the reconstructed field using those samples. It shows that the MC-ALM method is able to reconstruct the field with good fidelity. The simulation is conducted with Matlab on a desktop PC (Dell Vostro 460 with 3.1 GHz Intel i5-2400 CPU and 4 GB memory), and the elapsed CPU time is 0.4 s, obtained using the Matlab macro *cputime*.

For our proposed iterative adaptive sampling approach, 6 new samples are taken to recover



(a)



(b)

Figure 6.5: Reconstruction of field of interest using MC-ALM method. (a) sampled data; (b) reconstructed field.

the every new row or new column in each iteration, and thus 43.75% measurements are taken sequentially. The reconstructed data is identical to the original field as shown in Fig. 6.6 and the

CPU time for calculation is 0.04 s, which is 10% of that using the ALM-MC method.



Figure 6.6: Reconstruction of field of interest using the proposed method.

### 6.5.3 Comparisons of energy cost for data sampling

In order to evaluate the energy cost of each sampling strategy, consider a simplified energy model for robotic fish:

$$E = \sum_{j=1}^{n} \left( \kappa_1 \sum_{i=1}^{m_j} L_i + \kappa_2 m_j \right)$$
(6.4)

where E is the expended energy,  $\kappa_1$  and  $\kappa_2$  are coefficients that imply the linear motion cost and measurement cost, respectively.  $L_i$  is the moving distance of step *i*, and  $m_j$  is the number of measurements of the *j*th robotic moving platform. Here we neglect other costs, e.g., wireless communication cost, cost to change the motion direction of the robot, etc., to simplify the problem. By setting  $\kappa_1$  and  $\kappa_2$  with different values, we can simulate different scenarios. For instance, if  $\kappa_1 \gg \kappa_2$ , it implies the movement cost is very expensive while the measurement cost is cheap and negligible, and then the sampling strategy should drive the robots to have shorter travels. On the other hand, if  $\kappa_1 \ll \kappa_2$ , the sampling strategy should minimize the number of samplings to reduce the total cost. In this work, we primarily consider the first scenario, because that locomotion is the biggest source of energy expenditure for autonomous robotic fish [1].

For the MC-ALM approach, the question is given a set of sampling locations, what is the shortest possible route that visits every sampling point and returns to the initial point. In the simulation, we set n = 2, to denote that the sampling algorithm is conducted with 2 robots. It is a NP-complete multiple traveling salesman problem (MTSP) and its solution is shown in Fig. 6.4. The total traveling distance of the two robots is 382 inches after about 50,000 iterations using a genetic algorithm.

On the other hand, the movement planning of robots using our proposed adaptive sampling framework is a natural result when determining the next sampling points at every time step, and the results are shown in Fig. 6.8. The total traveling distance (including back to the initial locations - a direct line back to the initial point) is 290 inches, which is 24% less than that using the MC-ALM approach.



Figure 6.7: Robot paths to cover all the sampling points using the multiple traveling salesman method. The red line and the blue denotes the paths of the two robots.

## 6.6 Discussions

### 6.6.1 Impact of more accurate energy model

The current work uses the distance to approximate the movement energy cost of the robotic fish system. In reality, the cost is dependent on not only the distance but also the effort for changing the direction. Compared with Eq. (6.5), a more accurate energy model is given below:

$$E = \sum_{j=1}^{n} \left( \kappa_1 \sum_{i=1}^{m_j} L_i + \kappa_2 m_j + \kappa_3 \sum_{i=1}^{m_j} n(\Delta \theta_i)^2 \right),$$
(6.5)



Figure 6.8: Robot paths to cover all the sampling points using our proposed adaptive sampling algorithm. The red line and the blue denotes the paths of two robots.

where  $\kappa_3$  is the coefficient to denote the direction change cost and  $\Delta \theta_i$  is the magnitude of the direction change in step *i*. Using this new model for the cost function, the path planning for the robotic sensor networks using our proposed successive sampling method is no longer intuitive and needed to be carefully addressed.

# 6.6.2 Limitation of the proposed matrix completion algorithm via successive reconstruction

If the rank of the initial sampling matrix is less than that of the full field, then in each iteration, the rank of the new sub-matrix might be different from that of its immediate predecessor. The proposed algorithms in Section 6.4 will not work for this situation. In this part, we improve the proposed method as explained in Algorithm 3, with the cost of taking more samplings. We note that the extra samplings are picked randomly and there is no guarantee that a new linearly independent row or column (if existing) would be caught using Algorithm 3.

### Algorithm 3: Matrix Completion via Analytical Reconstruction

**Inputs**: Rank *r* of the original matrix *r* 

- 1 Sample the first  $r \times r$  matrix by initializing k = r
- 2 while the dimension of the recovered matrix is less than that of the original field do
- 3 // Lines 4-16 reconstruct M
- 4 Calculate the rank r' of M'
- 5 If r = r', go to Algorithm 2, else, continue to line 6
- 6 Find r' linearly independent columns as the basis  $B_{col}$
- 7 Take samples (on the outer row) in r' columns to form the new basis  $B'_{col}$  and another column  $v'_{col}$  randomly
- 8 If  $v'_{col}$  is independent of  $B'_{col}$ , sample every new points, and go to line 11
- 9 Express other columns *i* as a linear combination of the basis vectors  $B_{col}$  with  $(c_{i1}, \ldots c_{ir})^T$
- 10 Recover the outer row by multiplying coordinates with new basis  $B'_{col}$
- 11 Find r' linearly independent rows as the basis  $B_{row}$
- 12 Take samples (on the outer column) in r' rows to form the new basis  $B'_{row}$  and another row  $v'_{row}$  randomly
- 13 If  $v'_{row}$  is independent of  $B'_{row}$ , sample every new points, and go to step 16
- 14 Express other rows *j* (including the outer row) as a linear combination of the basis vectors  $B_{row}$  with  $(c_{i1}, \dots c_{ir})^T$
- 15 Recover the outer column by multiplying coordinates with new basis  $B'_{row}$
- 16 Update k to (k+1)
- 17 end while

Output: Recovered matrix.

To demonstrate the effectiveness of Algorithm 3, we generate a  $20 \times 20$  matrix (shown in Fig. 6.9) with rank 8, and its first  $10 \times 10$  sub-matrix has rank 7 and the first  $11 \times 11$  sub-matrix has rank 8.



Figure 6.9: Scalar field described by a  $20 \times 20$  matrix.

Using Algorithm 2, the reconstructed matrix is shown in Fig. 6.10, which has a big error compared with the original scalar field.

Fig. 6.11 shows the reconstructed field using Algorithm 3, which is able to perfectly recover the original data (matrix). Fig. 6.12 shows the sampling points when using Algorithm 3 to guide the robotic networks with the red stars denoting the additional samplings that needed to be taken compared with Algorithm 2.



Figure 6.10: Reconstruction of field of interest using Algorithm 2.

## 6.7 Conclusions

In this work, an adaptive sampling framework is proposed to reconstruct aquatic environmental fields using a group of robotic sensor platforms. In particular, it is assumed that the field of interest can be approximated by a low rank matrix, and then an iterative matrix completion approach via analytical solutions is proposed and explored. For comparison, an MC approach using Augmented Lagrange Multiplier optimization is also presented. Simulation results show that our proposed approach is more computationally efficient and requires shorter travel distances for the robots.

The future work can be extended in two directions. First, it will be of interest to explore ignoring weakly linearly independent columns in the rank computation and analyze the propagation of



Figure 6.11: Reconstruction of field of interest using Algorithm 3.

reconstruction error caused by this approximation, and the performance of the proposed analytical matrix reconstruction algorithm. Second, we will explore the validity of the approach using the robotic fish prototypes developed at the Smart Microsystems Lab.



Figure 6.12: Locations of samplings using Algorithm 3.

## **Chapter 7**

## **Conclusions and Future Work**

### 7.1 Conclusions

In this work, the design, development, modeling, control of robotic fish and an adaptive sampling framework have been studied.

First, the guidelines for robotic fish system design have been provided and demonstrated by presenting several robot prototypes. Applications with these robots, including aquatic environmental monitoring, robot-animal interaction study, education and outreach, are also presented.

Second, a complete dynamic model for robotic fish actuated by a caudal fin has been developed, where Lighthill's elongated-body theory is used to evaluate the tail-generated hydrodynamic force. It shows that incorporating the body motion in the evaluation of hydrodynamic forces, which is often ignored in literatures, plays a significant role in capturing the dynamics. Besides, simulation studies show that the drag force and moment coefficients are strongly dependent on the tail-beat bias, and subsequently a novel scheme is developed to adapt these coefficients and lift force using the parameters of tail movement.

Third, a model for robotic fish with a base-actuated flexible tail has been developed. The tail is modeled as multiple rigid segments connected by springs and dampers, and Lighthill's elongatedbody theory is used to evaluate the tail-generated hydrodynamic forces. Extensive experimental results have been collected, and for comparison, a model using linear beam theory is also constructed. From these results, it can be concluded that when the tail is excited under a relatively low frequency, and consequently experiences small deformation, both models produce similar predictions that are close to experimental measurements. However, when the actuation frequency increases, the two models differ, and the model using multi-segment approximation is able to predict much better the robot speed and the tail shape.

Fourth, a novel control-oriented data-driven averaging approach has been studied that can approximate well the original dynamics to provide better insight for the controller design. To demonstrate the merit of the proposed averaging approach, a semi-analytical framework for studying the robot's turning motion and a hybrid controller to make the robot follow a set of waypoints have been developed based upon the averaged model.

Last, an adaptive sampling framework for a group of robotic sensing platforms has been developed for the mapping of aquatic fields, where the original field is assumed to be represented by a low-rank matrix. The proposed method successfully expands the sampling area and computes the missing data points analytically. Real trace-driven simulation results show that the proposed approach is more computationally efficient and requires shorter travel distances than an established matrix completion scheme.

### 7.2 Future work

The future work can be extended in several directions. First, for the averaging of robotic fish dynamics, it is of interest to explore analytical insight as to why the scaling functions take the specific simple forms. Second, We will also seek to extend the approach to the dynamic model of robotic fish actuated with a flexible tail, as well as explore the validity of the approach for a wider class of systems with oscillatory inputs. Third, for the environmental reconstruction using the proposed successively spanning the sampling area strategy, it will be of interest to explore ignoring weakly linearly independent columns in the rank computation and analyze the propagation of reconstruction error caused by this approximation, and the performance of the proposed analytical matrix reconstruction algorithm. Forth, we will explore the validity of the approach using the robotic fish prototypes developed at the Smart Microsystems Lab.

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